JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

COMPUTER SCIENCE

VECTOR ANALYSIS

TIME: 2hrs

Q1

a) Determine **P** such that: $\bar{A} = 2\hat{\imath} + \hat{\jmath} + 4\hat{k}$, $\bar{B} = 3\hat{\imath} + 2\hat{\jmath} + P\hat{k}$, $\bar{C} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ are coplanar (4mks)

b) Find the unit tangent $\bar{T}$, the curvature K and the unit normal $\bar{N}$ for the curve $\vec{r} = -(3sin2t)\hat{\imath} - (3cos2t)\hat{\jmath}$ (7mks)

c) If $\emptyset = 3x^2y^2z$ find $divgrad\emptyset$ (4mks)

d) Prove that $\nabla^2(\ln r) = \frac{1}{r^2}$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ (6mks)

e) Find the work done by moving a particle once around a circle C in the xy plane if the circle has centre origin and radios 3 and the force field is given by

$$\vec{F} = (2x - y + z)\hat{\imath} + (x + y - z^2)\hat{\jmath} + (3x - 2y + 4z)\hat{k}$$

(5mks)

f) Evaluate the integral $\int_{x=0}^{a} \int_{y=0}^{\sqrt{a^2-x^2}} \int_{z=0}^{\sqrt{a^2-x^2}} dzdydx$ (4mks)

Q2

a) Determine the unit tangent vector to any point on the curve described by the parametric equations $x = t^2 + 1$, $y = 4t + 3$, $z = 2t^2 + 6t$ at the point $t = 2$ (6mks)

b) Find the curvature and unit normal to the space curve $\vec{r} = (t + 1)\hat{\imath} + \frac{\sqrt{6}}{3}t^3\hat{\jmath} + \frac{3}{5}t^5\hat{k}$ at $t = 1$ (14mks)

Q3

a) If $\vec{F} = xy\hat{\imath} - z\hat{\jmath} + x^2\hat{k}$ and C is the curve $x = t^2$ $y = 2t$ $z = t^2$ from $t = 1 \ to \ t = 2$, evaluate the integral $\oint_C \vec{F} \times d\vec{r}$ (5mks)

b) Evaluate $\iint_S \vec{B} \cdot \hat{n} ds$ where $\bar{B} = z\hat{\imath} + x\hat{\jmath} + 3y^2 z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 49$ included in the first octant between z=0 and z=6 (10mks)

c) Let $\vec{F} = 2xz\hat{\imath} - x\hat{\jmath} + y^2\hat{k}$. Evaluate $\iiint_v \vec{F} dv$ where V is the region bounded by the surfaces $x = 0$, $x = 1$, $y = 0$, $y = 6$ $z = x^2$ $and\ z = 9$ (5mks)

Q4

a) Prove that the cylindrical coordinate system is orthogonal (6mks)

b) State the Green's theorem hence verify the theorem in the plane for $\oint_C (2xy - x^2)dx + (x + y^2)dy$ where C is a closed curve of region bounded by $x = y^2$ $y = x$ (14mks)