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**University Examinations 2014/2015**

SECONDYEAR, SPECIAL/SUPPLEMENTARY EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS

**SMA 2202: ALGEBRAIC STRUCTURES**

**DATE: JULY 2015 TIME: 2 HOURS**

**INSTRUCTIONS:** *Answer questions* ***one*** *and any other* ***two*** *questions.*

**QUESTION ONE (30 MARKS)**

1. (i) Define an abelian group (2 marks)

(ii) Show that the set $Z$ of all integers under the binary operation $⋆$ defined by $a ⋆$ b, $a+b+2,∀a,b\in Z$ is an abelian group (4 marks)

1. Let and find  (3 marks)
2. Prove that  is a cyclic group generated by 1. (4 marks)
3. Given the and prove that  (4 marks)
4. Given that a,b,cand is a group. Prove that  (4 marks)
5. Show that the binary operation over$Z$, defined by commutative is but not associative. (4 marks)
6. (i) Show that the set is a commutative ring under addition modulo 8 (3 marks)

(ii) Show that the ring in (i) above is not an integral domain (2 marks)

**QUESTION TWO (20 MARKS)**

1. Let $f$ and $g$ be two permutations of $s\_{7}$where and find
2. $fg$ (5 marks)
3.  (5 marks)
4. List all the symmetries of an equilateral triangle 1

(6 marks)

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1. Let . The binary operation on G is given by the table below

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| 1 | 1 | 2 | 3 | 4 |
| 2 | 2 | 4 | 2 | 4 |
| 3 | 3 | 2 | 1 | 4 |
| 4 | 4 | 4 | 4 | 4 |

1. Find the identity element in G (1 mark)
2. Which elements have inverses? (2 marks)
3. Are the inverses unique? (1 mark)

**QUESTION THREE (20 MARKS)**

1. Prove that the set of non-zero integers modulo 6 under $x\_{6} $is not a group (8 marks)
2. Show that if is a group, then the unique solution of the equation is  (4 marks)
3. Show that is a cyclic group generated by 3 and 2 only (8 marks)

**QUESTION FOUR (20 MARKS)**

1. Given that is a subgroup of the group . Define a left coset of H in G. (3 marks)
2. Show that the left cosets of decomposes $Z\_{6},+6$into two disjoint sets (6 marks)
3. Prove that the order of each subgroup of a finite group divides the order of the group (7 marks)
4. Consider the binary operation table defined over a set 

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | a | b | c | d |
| a | a | b | c | d |
| b | b | a | c | d |
| c | c | d | c | d |
| d | d | c | c | d |

Determine whatever in the table satisfies:

1. Commutative law (2 marks)
2. Associative law (2 marks)

**QUESTION FIVE (20 MARKS)**

Prove that the set G composed of of the 4 transformations of the set C of complex numbers on itself defined by and is an abelian group with the composite operation i.e where and  (20 marks)