**KENYA METHODIST UNIVERSITY**

**END OF 1ST TRIMESTER 2018 (FT) EXAMINATION**

**SCHOOL : SCIENCE & TECHNOLOGY**

**DEPARTMENT : PURE AND APPLIED SCIENCES**

**UNIT CODE : MATH 230**

**UNIT TITLE : PROBABILITY AND STATISTICS II**

**TIME : 2 HOURS**

**INSTRUCTIONS**

* ***Answer Question One and any other two questions.***

**Question One:**

1. Distinguish between type I and type II error as used in tests of hypothesis. (2 marks)
2. Determine whether the function given by  for *x* = 1, 2, 3, 4, 5 can serve as a probability distribution of a discrete random variable. (4 marks)
3. Suppose that the error in the reaction temperature for a controlled laboratory experiment is a continuous random variable $X$ having the probability density function

$f\left(x\right)=\left\{\begin{array}{c}\frac{x^{2}}{3}, -1<x<2\\0 elsewhere\end{array}\right. $ Find the probability $P(0<X\leq 1)$ (4 marks)

1. An electrical firm manufactures light bulbs that have a length of life that is normally distributed with mean equal to 800 hours and standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. (4 marks)
2. compute the standard deviation of the random variable defined as follows: (4 marks)

|  |  |  |  |
| --- | --- | --- | --- |
| xi | 100 | 140 | 210 |
|  pi | 0.4 | 0.5 | 0.1 |

1. Show that if a random variable X is uniformly distributed over [a,b] its mean E(X) is given by (4 marks)

$$\frac{a+b}{2}$$

1. The life expectancy of people in Kenya in the year 2020 is expected to be 50 years. A survey was conducted in 11 counties of Kenya and the data obtained is given below.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| county | A | B | C | D | E | F | G | H | I | J | K |
| Life expectancy | 54.2 | 50.4 | 44.2 | 49.7 | 55.4 | 57.0 | 58.2 | 56.6 | 61.9 | 57.5 | 53.4 |

Test at 0.1 level of significance to whether the data confirms the expected view. (8 marks)

**Question Two:**

1. By citing an example in each case, Distinguish between a probability mass function (p.m.f) and a probability density function (p.d.f) (4 marks)
2. A shipment of 20 similar laptop computers to a retail outlet contains three that are defective. If a school makes random purchases of three of these computers, find the probability distribution for the number of defectives. (4 marks)
3. The length of time *Y*, in minutes, required to generate a human reflex to tear gas has the density function

$$f\left(y\right)=\left\{\begin{array}{c}\frac{1}{4}\\0, \end{array}e^{\frac{-y}{4}}\right.,0\leq y\leq \infty $$

1. What is the mean time to reflex? (4 marks)
2. (Find the standard deviation of the random variable. (4 marks)
3. Find the distribution function of the random variable *X* whose probability density is given by



And use it to evaluate the probability  (4 marks)

**Question Three:**

a) It is expected that 10% of production from a continuous process will be defective. Find the probability that in a sample of 10 units chosen at random;

1. Exactly 2 will be defective (3 marks)
2. At least 2 will be defective. (5 marks)

b) A box of 20 spare parts for a certain type of a machine contains 15 good items and 5 defective items. If 4 parts are selected by chance from the box, what is the probability that exactly 3 of them will be good? (4 marks)

c) The number of calls per 10 minutes received at a telephone switch board follows a Poisson distribution with mean 0.6.Find the probability that:

1. No call will be received in the first 10 minutes. (3 marks)
2. More than 2 calls will be received in a period of 40 minutes. (5 marks)

**Question Four:**

1. Distinguish between parametric and non-parametric statistical test. Give an example in each case. (4 marks)
2. A manger wants to see if geographical region is associated with ownership of a Macintosh computer. The manager surveys 100 people and the data breaks down as follows: test at 5% whether ownership of a mac is related to geographical region. (6 marks)

|  |  |  |  |
| --- | --- | --- | --- |
|  | Mac | No Mac | Row total |
| North East | 12 | 14 | 26 |
| South West | 21 | 18 | 39 |
| Mid-West | 17 | 18 | 35 |
| Column Total | 50 | 50 | 100 |

1. A manufacturer of machine parts is considering buying one of the three machines currently in the market. The following is their daily output for five different days;

|  |
| --- |
| machine |
| A | B | C |
| 25 | 31 | 24 |
| 30 | 39 | 30 |
| 36 | 38 | 28 |
| 38 | 42 | 25 |
| 31 | 35 | 28 |

Do the machines have equal output rate? Test at α=0.05 (10 marks)

**TABLE A.1**

**Cumulative Standardized Normal Distribution**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | *A*(*z*) |  |  |
| -4 | -3 | -2 | -1 | 0 | 1 *z* 2 | 3 | 4 |



*A*(*z*) is the integral of the standardized normaldistribution from − ∞ to *z* (in other words, the area under the curve to the left of *z*). It gives the probability of a normal random variable not being more than *z* standard deviations above its mean. Values of *z* of particular importance:

|  |  |  |
| --- | --- | --- |
| *z* | *A*(*z*) |  |
| 1.645 | 0.9500 | Lower limit of right 5% tail |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail |
| 2.326 | 0.9900 | Lower limit of right 1% tail |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *z* | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|  |  |  |  |  |  |  |  |  |  |  |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

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| STATISTICAL TABLES |  |  |  |  |  | **2** |
|  |  |  | **TABLE A.2** |  |  |  |  |
|  |  | ***t* Distribution: Critical Values of *t*** |  |  |  |
|  |  |  |  | *Significance level* |  |  |
| *Degrees of* | *Two-tailed test*: | 10% | 5% | 2% | 1% | 0.2% | 0.1% |
| *freedom* | *One-tailed test*: | 5% | 2.5% | 1% | 0.5% | 0.1% | 0.05% |
| **1** |  | 6.314 | 12.706 | 31.821 | 63.657 | 318.309 | 636.619 |
| **2** |  | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| **3** |  | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| **4** |  | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| **5** |  | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| **6** |  | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| **7** |  | 1.894 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| **8** |  | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| **9** |  | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| **10** |  | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| **11** |  | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| **12** |  | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| **13** |  | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| **14** |  | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| **15** |  | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| **16** |  | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| **17** |  | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| **18** |  | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| **19** |  | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| **20** |  | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| **21** |  | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| **22** |  | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| **23** |  | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| **24** |  | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| **25** |  | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| **26** |  | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| **27** |  | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| **28** |  | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| **29** |  | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 |
| **30** |  | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| **32** |  | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 | 3.622 |
| **34** |  | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 | 3.601 |
| **36** |  | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 | 3.582 |
| **38** |  | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 | 3.566 |
| **40** |  | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| **42** |  | 1.682 | 2.018 | 2.418 | 2.698 | 3.296 | 3.538 |
| **44** |  | 1.680 | 2.015 | 2.414 | 2.692 | 3.286 | 3.526 |
| **46** |  | 1.679 | 2.013 | 2.410 | 2.687 | 3.277 | 3.515 |
| **48** |  | 1.677 | 2.011 | 2.407 | 2.682 | 3.269 | 3.505 |
| **50** |  | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 | 3.496 |
| **60** |  | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| **70** |  | 1.667 | 1.994 | 2.381 | 2.648 | 3.211 | 3.435 |
| **80** |  | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| **90** |  | 1.662 | 1.987 | 2.368 | 2.632 | 3.183 | 3.402 |
| **100** |  | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| **120** |  | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 |
| **150** |  | 1.655 | 1.976 | 2.351 | 2.609 | 3.145 | 3.357 |
| **200** |  | 1.653 | 1.972 | 2.345 | 2.601 | 3.131 | 3.340 |
| **300** |  | 1.650 | 1.968 | 2.339 | 2.592 | 3.118 | 3.323 |
| **400** |  | 1.649 | 1.966 | 2.336 | 2.588 | 3.111 | 3.315 |
| **500** |  | 1.648 | 1.965 | 2.334 | 2.586 | 3.107 | 3.310 |
| **600** |  | 1.647 | 1.964 | 2.333 | 2.584 | 3.104 | 3.307 |
| ∞ |  | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |

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| STATISTICAL TABLES | **3** |

**TABLE A.3**

***F* Distribution: Critical Values of *F* (5% significance level)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ***v*1** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **12** | **14** | **16** | **18** | **20** |  |
| ***v*2** | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 243.91 | 245.36 | 246.46 | 247.32 | 248.01 |  |
| **1** |  |
| **2** | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.42 | 19.43 | 19.44 | 19.45 |  |
| **3** | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.71 | 8.69 | 8.67 | 8.66 |  |
| **4** | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.87 | 5.84 | 5.82 | 5.80 |  |
| **5** | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.64 | 4.60 | 4.58 | 4.56 |  |
| **6** | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.96 | 3.92 | 3.90 | 3.87 |  |
| **7** | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.53 | 3.49 | 3.47 | 3.44 |  |
| **8** | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.24 | 3.20 | 3.17 | 3.15 |  |
| **9** | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.03 | 2.99 | 2.96 | 2.94 |  |
| **10** | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.86 | 2.83 | 2.80 | 2.77 |  |
| **11** | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.74 | 2.70 | 2.67 | 2.65 |  |
| **12** | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.64 | 2.60 | 2.57 | 2.54 |  |
| **13** | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.55 | 2.51 | 2.48 | 2.46 |  |
| **14** | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.48 | 2.44 | 2.41 | 2.39 |  |
| **15** | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.42 | 2.38 | 2.35 | 2.33 |  |
| **16** | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.37 | 2.33 | 2.30 | 2.28 |  |
| **17** | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.33 | 2.29 | 2.26 | 2.23 |  |
| **18** | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.29 | 2.25 | 2.22 | 2.19 |  |
| **19** | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.26 | 2.21 | 2.18 | 2.16 |  |
| **20** | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.22 | 2.18 | 2.15 | 2.12 |  |
| **21** | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.20 | 2.16 | 2.12 | 2.10 |  |
| **22** | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.17 | 2.13 | 2.10 | 2.07 |  |
| **23** | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.15 | 2.11 | 2.08 | 2.05 |  |
| **24** | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.13 | 2.09 | 2.05 | 2.03 |  |
| **25** | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.11 | 2.07 | 2.04 | 2.01 |  |
| **26** | 4.22 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.15 | 2.09 | 2.05 | 2.02 | 1.99 |  |
| **27** | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 | 1.97 |  |
| **28** | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.12 | 2.06 | 2.02 | 1.99 | 1.96 |  |
| **29** | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.10 | 2.05 | 2.01 | 1.97 | 1.94 |  |
| **30** | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.09 | 2.04 | 1.99 | 1.96 | 1.93 |  |
| **35** | 4.12 | 3.27 | 2.87 | 2.64 | 2.49 | 2.37 | 2.29 | 2.22 | 2.16 | 2.11 | 2.04 | 1.99 | 1.94 | 1.91 | 1.88 |  |
| **40** | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.00 | 1.95 | 1.90 | 1.87 | 1.84 |  |
| **50** | 4.03 | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 | 2.03 | 1.95 | 1.89 | 1.85 | 1.81 | 1.78 |  |
| **60** | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.92 | 1.86 | 1.82 | 1.78 | 1.75 |  |
| **70** | 3.98 | 3.13 | 2.74 | 2.50 | 2.35 | 2.23 | 2.14 | 2.07 | 2.02 | 1.97 | 1.89 | 1.84 | 1.79 | 1.75 | 1.72 |  |
| **80** | 3.96 | 3.11 | 2.72 | 2.49 | 2.33 | 2.21 | 2.13 | 2.06 | 2.00 | 1.95 | 1.88 | 1.82 | 1.77 | 1.73 | 1.70 |  |
| **90** | 3.95 | 3.10 | 2.71 | 2.47 | 2.32 | 2.20 | 2.11 | 2.04 | 1.99 | 1.94 | 1.86 | 1.80 | 1.76 | 1.72 | 1.69 |  |
| **100** | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.85 | 1.79 | 1.75 | 1.71 | 1.68 |  |
| **120** | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 | 1.91 | 1.83 | 1.78 | 1.73 | 1.69 | 1.66 |  |
| **150** | 3.90 | 3.06 | 2.66 | 2.43 | 2.27 | 2.16 | 2.07 | 2.00 | 1.94 | 1.89 | 1.82 | 1.76 | 1.71 | 1.67 | 1.64 |  |
| **200** | 3.89 | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 | 1.88 | 1.80 | 1.74 | 1.69 | 1.66 | 1.62 |  |
| **250** | 3.88 | 3.03 | 2.64 | 2.41 | 2.25 | 2.13 | 2.05 | 1.98 | 1.92 | 1.87 | 1.79 | 1.73 | 1.68 | 1.65 | 1.61 |  |
| **300** | 3.87 | 3.03 | 2.63 | 2.40 | 2.24 | 2.13 | 2.04 | 1.97 | 1.91 | 1.86 | 1.78 | 1.72 | 1.68 | 1.64 | 1.61 |  |
| **400** | 3.86 | 3.02 | 2.63 | 2.39 | 2.24 | 2.12 | 2.03 | 1.96 | 1.90 | 1.85 | 1.78 | 1.72 | 1.67 | 1.63 | 1.60 |  |
| **500** | 3.86 | 3.01 | 2.62 | 2.39 | 2.23 | 2.12 | 2.03 | 1.96 | 1.90 | 1.85 | 1.77 | 1.71 | 1.66 | 1.62 | 1.59 |  |
| **600** | 3.86 | 3.01 | 2.62 | 2.39 | 2.23 | 2.11 | 2.02 | 1.95 | 1.90 | 1.85 | 1.77 | 1.71 | 1.66 | 1.62 | 1.59 |  |
| **750** | 3.85 | 3.01 | 2.62 | 2.38 | 2.23 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 | 1.77 | 1.70 | 1.66 | 1.62 | 1.58 |  |
| **1000** | 3.85 | 3.00 | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 | 1.76 | 1.70 | 1.65 | 1.61 | 1.58 |  |

STATISTICAL TABLES

**TABLE A.4**

*χ***2 (Chi-Squared) Distribution: Critical Values of** *χ***2**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | *Significance level* |  |
| *Degrees of* | 5% | 1% | 0.1% |
| *freedom* |  |  |  |
| **1** | 3.841 | 6.635 | 10.828 |
| **2** | 5.991 | 9.210 | 13.816 |
| **3** | 7.815 | 11.345 | 16.266 |
| **4** | 9.488 | 13.277 | 18.467 |
| **5** | 11.070 | 15.086 | 20.515 |
| **6** | 12.592 | 16.812 | 22.458 |
| **7** | 14.067 | 18.475 | 24.322 |
| **8** | 15.507 | 20.090 | 26.124 |
| **9** | 16.919 | 21.666 | 27.877 |
| **10** | 18.307 | 23.209 | 29.588 |