**KENYA METHODIST UNIVERSITY**

**END OF 1ST TRIMESTER 2018 (FT) EXAMINATION**

**SCHOOL : SCIENCE & TECHNOLOGY**

**DEPARTMENT : PURE AND APPLIED SCIENCES**

**UNIT CODE : MATH 230**

**UNIT TITLE : PROBABILITY AND STATISTICS II**

**TIME : 2 HOURS**

**INSTRUCTIONS**

* ***Answer Question One and any other two questions.***

**Question One:**

1. Distinguish between type I and type II error as used in tests of hypothesis. (2 marks)
2. Determine whether the function given by  for *x* = 1, 2, 3, 4, 5 can serve as a probability distribution of a discrete random variable. (4 marks)
3. Suppose that the error in the reaction temperature for a controlled laboratory experiment is a continuous random variable having the probability density function

Find the probability (4 marks)

1. An electrical firm manufactures light bulbs that have a length of life that is normally distributed with mean equal to 800 hours and standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. (4 marks)
2. compute the standard deviation of the random variable defined as follows: (4 marks)

|  |  |  |  |
| --- | --- | --- | --- |
| xi | 100 | 140 | 210 |
| pi | 0.4 | 0.5 | 0.1 |

1. Show that if a random variable X is uniformly distributed over [a,b] its mean E(X) is given by (4 marks)
2. The life expectancy of people in Kenya in the year 2020 is expected to be 50 years. A survey was conducted in 11 counties of Kenya and the data obtained is given below.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| county | A | B | C | D | E | F | G | H | I | J | K |
| Life expectancy | 54.2 | 50.4 | 44.2 | 49.7 | 55.4 | 57.0 | 58.2 | 56.6 | 61.9 | 57.5 | 53.4 |

Test at 0.1 level of significance to whether the data confirms the expected view. (8 marks)

**Question Two:**

1. By citing an example in each case, Distinguish between a probability mass function (p.m.f) and a probability density function (p.d.f) (4 marks)
2. A shipment of 20 similar laptop computers to a retail outlet contains three that are defective. If a school makes random purchases of three of these computers, find the probability distribution for the number of defectives. (4 marks)
3. The length of time *Y*, in minutes, required to generate a human reflex to tear gas has the density function
4. What is the mean time to reflex? (4 marks)
5. (Find the standard deviation of the random variable. (4 marks)
6. Find the distribution function of the random variable *X* whose probability density is given by



And use it to evaluate the probability  (4 marks)

**Question Three:**

a) It is expected that 10% of production from a continuous process will be defective. Find the probability that in a sample of 10 units chosen at random;

1. Exactly 2 will be defective (3 marks)
2. At least 2 will be defective. (5 marks)

b) A box of 20 spare parts for a certain type of a machine contains 15 good items and 5 defective items. If 4 parts are selected by chance from the box, what is the probability that exactly 3 of them will be good? (4 marks)

c) The number of calls per 10 minutes received at a telephone switch board follows a Poisson distribution with mean 0.6.Find the probability that:

1. No call will be received in the first 10 minutes. (3 marks)
2. More than 2 calls will be received in a period of 40 minutes. (5 marks)

**Question Four:**

1. Distinguish between parametric and non-parametric statistical test. Give an example in each case. (4 marks)
2. A manger wants to see if geographical region is associated with ownership of a Macintosh computer. The manager surveys 100 people and the data breaks down as follows: test at 5% whether ownership of a mac is related to geographical region. (6 marks)

|  |  |  |  |
| --- | --- | --- | --- |
|  | Mac | No Mac | Row total |
| North East | 12 | 14 | 26 |
| South West | 21 | 18 | 39 |
| Mid-West | 17 | 18 | 35 |
| Column Total | 50 | 50 | 100 |

1. A manufacturer of machine parts is considering buying one of the three machines currently in the market. The following is their daily output for five different days;

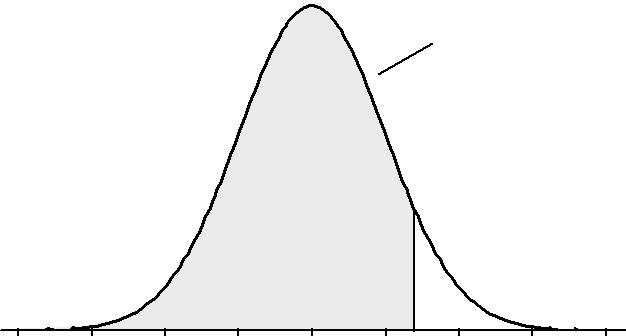
|  |  |  |
| --- | --- | --- |
| machine | | |
| A | B | C |
| 25 | 31 | 24 |
| 30 | 39 | 30 |
| 36 | 38 | 28 |
| 38 | 42 | 25 |
| 31 | 35 | 28 |

Do the machines have equal output rate? Test at α=0.05 (10 marks)

**TABLE A.1**

**Cumulative Standardized Normal Distribution**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | *A*(*z*) |  |  |
| -4 | -3 | -2 | -1 | 0 | 1 *z* 2 | 3 | 4 |



*A*(*z*) is the integral of the standardized normaldistribution from − ∞ to *z* (in other words, the area under the curve to the left of *z*). It gives the probability of a normal random variable not being more than *z* standard deviations above its mean. Values of *z* of particular importance:

|  |  |  |
| --- | --- | --- |
| *z* | *A*(*z*) |  |
| 1.645 | 0.9500 | Lower limit of right 5% tail |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail |
| 2.326 | 0.9900 | Lower limit of right 1% tail |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *z* | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|  |  |  |  |  |  |  |  |  |  |  |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

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| --- | --- | --- | --- | --- | --- | --- | --- |
| STATISTICAL TABLES | |  |  |  |  |  | **2** |
|  |  |  | **TABLE A.2** |  |  |  |  |
|  |  | ***t* Distribution: Critical Values of *t*** | | |  |  |  |
|  |  |  |  | *Significance level* | |  |  |
| *Degrees of* | *Two-tailed test*: | 10% | 5% | 2% | 1% | 0.2% | 0.1% |
| *freedom* | *One-tailed test*: | 5% | 2.5% | 1% | 0.5% | 0.1% | 0.05% |
| **1** |  | 6.314 | 12.706 | 31.821 | 63.657 | 318.309 | 636.619 |
| **2** |  | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| **3** |  | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| **4** |  | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| **5** |  | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| **6** |  | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| **7** |  | 1.894 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| **8** |  | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| **9** |  | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| **10** |  | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| **11** |  | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| **12** |  | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| **13** |  | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| **14** |  | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| **15** |  | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| **16** |  | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| **17** |  | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| **18** |  | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| **19** |  | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| **20** |  | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| **21** |  | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| **22** |  | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| **23** |  | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| **24** |  | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| **25** |  | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| **26** |  | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| **27** |  | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| **28** |  | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| **29** |  | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 |
| **30** |  | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| **32** |  | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 | 3.622 |
| **34** |  | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 | 3.601 |
| **36** |  | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 | 3.582 |
| **38** |  | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 | 3.566 |
| **40** |  | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| **42** |  | 1.682 | 2.018 | 2.418 | 2.698 | 3.296 | 3.538 |
| **44** |  | 1.680 | 2.015 | 2.414 | 2.692 | 3.286 | 3.526 |
| **46** |  | 1.679 | 2.013 | 2.410 | 2.687 | 3.277 | 3.515 |
| **48** |  | 1.677 | 2.011 | 2.407 | 2.682 | 3.269 | 3.505 |
| **50** |  | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 | 3.496 |
| **60** |  | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| **70** |  | 1.667 | 1.994 | 2.381 | 2.648 | 3.211 | 3.435 |
| **80** |  | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| **90** |  | 1.662 | 1.987 | 2.368 | 2.632 | 3.183 | 3.402 |
| **100** |  | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| **120** |  | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 |
| **150** |  | 1.655 | 1.976 | 2.351 | 2.609 | 3.145 | 3.357 |
| **200** |  | 1.653 | 1.972 | 2.345 | 2.601 | 3.131 | 3.340 |
| **300** |  | 1.650 | 1.968 | 2.339 | 2.592 | 3.118 | 3.323 |
| **400** |  | 1.649 | 1.966 | 2.336 | 2.588 | 3.111 | 3.315 |
| **500** |  | 1.648 | 1.965 | 2.334 | 2.586 | 3.107 | 3.310 |
| **600** |  | 1.647 | 1.964 | 2.333 | 2.584 | 3.104 | 3.307 |
| ∞ |  | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |

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| --- | --- |
| STATISTICAL TABLES | **3** |

**TABLE A.3**

***F* Distribution: Critical Values of *F* (5% significance level)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ***v*1** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **12** | **14** | **16** | **18** | **20** |  |
| ***v*2** | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 243.91 | 245.36 | 246.46 | 247.32 | 248.01 |  |
| **1** |  |
| **2** | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.42 | 19.43 | 19.44 | 19.45 |  |
| **3** | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.71 | 8.69 | 8.67 | 8.66 |  |
| **4** | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.87 | 5.84 | 5.82 | 5.80 |  |
| **5** | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.64 | 4.60 | 4.58 | 4.56 |  |
| **6** | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.96 | 3.92 | 3.90 | 3.87 |  |
| **7** | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.53 | 3.49 | 3.47 | 3.44 |  |
| **8** | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.24 | 3.20 | 3.17 | 3.15 |  |
| **9** | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.03 | 2.99 | 2.96 | 2.94 |  |
| **10** | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.86 | 2.83 | 2.80 | 2.77 |  |
| **11** | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.74 | 2.70 | 2.67 | 2.65 |  |
| **12** | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.64 | 2.60 | 2.57 | 2.54 |  |
| **13** | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.55 | 2.51 | 2.48 | 2.46 |  |
| **14** | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.48 | 2.44 | 2.41 | 2.39 |  |
| **15** | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.42 | 2.38 | 2.35 | 2.33 |  |
| **16** | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.37 | 2.33 | 2.30 | 2.28 |  |
| **17** | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.33 | 2.29 | 2.26 | 2.23 |  |
| **18** | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.29 | 2.25 | 2.22 | 2.19 |  |
| **19** | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.26 | 2.21 | 2.18 | 2.16 |  |
| **20** | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.22 | 2.18 | 2.15 | 2.12 |  |
| **21** | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.20 | 2.16 | 2.12 | 2.10 |  |
| **22** | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.17 | 2.13 | 2.10 | 2.07 |  |
| **23** | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.15 | 2.11 | 2.08 | 2.05 |  |
| **24** | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.13 | 2.09 | 2.05 | 2.03 |  |
| **25** | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.11 | 2.07 | 2.04 | 2.01 |  |
| **26** | 4.22 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.15 | 2.09 | 2.05 | 2.02 | 1.99 |  |
| **27** | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 | 1.97 |  |
| **28** | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.12 | 2.06 | 2.02 | 1.99 | 1.96 |  |
| **29** | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.10 | 2.05 | 2.01 | 1.97 | 1.94 |  |
| **30** | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.09 | 2.04 | 1.99 | 1.96 | 1.93 |  |
| **35** | 4.12 | 3.27 | 2.87 | 2.64 | 2.49 | 2.37 | 2.29 | 2.22 | 2.16 | 2.11 | 2.04 | 1.99 | 1.94 | 1.91 | 1.88 |  |
| **40** | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.00 | 1.95 | 1.90 | 1.87 | 1.84 |  |
| **50** | 4.03 | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 | 2.03 | 1.95 | 1.89 | 1.85 | 1.81 | 1.78 |  |
| **60** | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.92 | 1.86 | 1.82 | 1.78 | 1.75 |  |
| **70** | 3.98 | 3.13 | 2.74 | 2.50 | 2.35 | 2.23 | 2.14 | 2.07 | 2.02 | 1.97 | 1.89 | 1.84 | 1.79 | 1.75 | 1.72 |  |
| **80** | 3.96 | 3.11 | 2.72 | 2.49 | 2.33 | 2.21 | 2.13 | 2.06 | 2.00 | 1.95 | 1.88 | 1.82 | 1.77 | 1.73 | 1.70 |  |
| **90** | 3.95 | 3.10 | 2.71 | 2.47 | 2.32 | 2.20 | 2.11 | 2.04 | 1.99 | 1.94 | 1.86 | 1.80 | 1.76 | 1.72 | 1.69 |  |
| **100** | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.85 | 1.79 | 1.75 | 1.71 | 1.68 |  |
| **120** | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 | 1.91 | 1.83 | 1.78 | 1.73 | 1.69 | 1.66 |  |
| **150** | 3.90 | 3.06 | 2.66 | 2.43 | 2.27 | 2.16 | 2.07 | 2.00 | 1.94 | 1.89 | 1.82 | 1.76 | 1.71 | 1.67 | 1.64 |  |
| **200** | 3.89 | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 | 1.88 | 1.80 | 1.74 | 1.69 | 1.66 | 1.62 |  |
| **250** | 3.88 | 3.03 | 2.64 | 2.41 | 2.25 | 2.13 | 2.05 | 1.98 | 1.92 | 1.87 | 1.79 | 1.73 | 1.68 | 1.65 | 1.61 |  |
| **300** | 3.87 | 3.03 | 2.63 | 2.40 | 2.24 | 2.13 | 2.04 | 1.97 | 1.91 | 1.86 | 1.78 | 1.72 | 1.68 | 1.64 | 1.61 |  |
| **400** | 3.86 | 3.02 | 2.63 | 2.39 | 2.24 | 2.12 | 2.03 | 1.96 | 1.90 | 1.85 | 1.78 | 1.72 | 1.67 | 1.63 | 1.60 |  |
| **500** | 3.86 | 3.01 | 2.62 | 2.39 | 2.23 | 2.12 | 2.03 | 1.96 | 1.90 | 1.85 | 1.77 | 1.71 | 1.66 | 1.62 | 1.59 |  |
| **600** | 3.86 | 3.01 | 2.62 | 2.39 | 2.23 | 2.11 | 2.02 | 1.95 | 1.90 | 1.85 | 1.77 | 1.71 | 1.66 | 1.62 | 1.59 |  |
| **750** | 3.85 | 3.01 | 2.62 | 2.38 | 2.23 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 | 1.77 | 1.70 | 1.66 | 1.62 | 1.58 |  |
| **1000** | 3.85 | 3.00 | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 | 1.76 | 1.70 | 1.65 | 1.61 | 1.58 |  |

STATISTICAL TABLES

**TABLE A.4**

*χ***2 (Chi-Squared) Distribution: Critical Values of** *χ***2**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | *Significance level* |  |
| *Degrees of* | 5% | 1% | 0.1% |
| *freedom* |  |  |  |
| **1** | 3.841 | 6.635 | 10.828 |
| **2** | 5.991 | 9.210 | 13.816 |
| **3** | 7.815 | 11.345 | 16.266 |
| **4** | 9.488 | 13.277 | 18.467 |
| **5** | 11.070 | 15.086 | 20.515 |
| **6** | 12.592 | 16.812 | 22.458 |
| **7** | 14.067 | 18.475 | 24.322 |
| **8** | 15.507 | 20.090 | 26.124 |
| **9** | 16.919 | 21.666 | 27.877 |
| **10** | 18.307 | 23.209 | 29.588 |