MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2017/2018

FIRST YEAR SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN
INFORMATION TECHNOLOGY

CITY CAMPUS

CIT 106: LINEAR ALGEBRA

Date: 16th June, 2018 Time: 8.00 - 11.00am

INSTRUCTIONS:

- Answer Question ONE and any other TWO.

QUESTION ONE (30 MARKS)

a) Differentiate between vector space and a vector subspace. (4 marks)

b) Define the following terms as used in Algebra (6 marks)

i. Adjoint matrix

ii. Linearly dependent vectors

iii. Spanning vectors

c) Show that the points A (-2,-2), B (2, 1) and C (10, 7) are collinear. (4 marks)

d) i. State the ratio theorem.

ii. Use the ratio theorem to find the position vector of point R which divides PQ externally in the ratio 5:2.

(4 marks)

e) Use the Cramer's Rule to solve the following system of equations;

$$x_1 + 2x_2 + 3x_3 = 3$$
$$2x_1 + 4x_2 + 5x_3 = 4$$
$$3x_1 + 5x_2 + 6x_3 = 8$$

(6 marks)

f) Find the Eigen values and Eigen vectors of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$.

(8 marks)

QUESTION TWO (20 MARKS)

a) Use the Gaus-Jordan method to solve the following system of equations;

$$x_1 + 2x_2 + x_3 = 6$$
$$2x_1 + 3x_2 + 4x_3 = 12$$
$$3x_1 + x_2 + 2x_3 = 7$$

(7 marks)

b) Let M_{22} be the collection of 2x2 matrices be a vector space. Show that the set W of all 2x2 matrices having zeros on the main diagonal is a subspace of M_{22}.

(4 marks)

c) i. Determine whether $\vec{v_1} = (1, 1, 2)$, $\vec{v_2} = (1, 0, 1)$ and $\vec{v_3} = (2, 1, 3)$ span $\mathbb{R}^3$.

(5 marks)

ii. Show that the vectors $\vec{i} = (1, 0, 0)$, $\vec{j} = (0, 1, 0)$, and $\vec{k} = (0, 0, 1)$ span $\mathbb{R}^3$.

(4 marks)

QUESTION THREE (20 MARKS)

a) Differentiate between a diagonal matrix and an identity matrix. (4 marks)

b) Let $\vec{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 15 \\ 3 \end{pmatrix}$, and $\vec{c} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$. Express $\vec{a}$ as a linear combination of $\vec{b}$ and $\vec{c}$. (4 marks)

c) i. Find the inverse of $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}$.

ii. Use the results in (i) above solve the following system of equations;

$$x_1 + x_2 + x_3 = 4$$
$$2x_1 + x_3 = 1$$
$$x_1 + 2x_2 = 2$$

(12 marks)

QUESTION FOUR (20 MARKS)

a) Define the following terms as used in Algebra

i. A matrix

ii. Linearly independent vectors

iii. A symmetric matrix

iv. An anti-symmetric matrix (8 marks)

b) Use the ratio theorem to find the position vector of point R which divides PQ internally in the ratio 4:3. (3 marks)

c) i. Show that the vectors;
$\vec{v_1} = (1, 0, 1, 2)$, $\vec{v_2} = (0, 1, 1, 2)$ and $\vec{v_3} = (1, 1, 1, 3)$ are linearly independent.

ii. Show that the vectors;
$\vec{v_1} = (1, 2, -1)$, $\vec{v_2} = (1, 2, -1)$ and $\vec{v_3} = (1, -2, 1)$ are linearly dependent.

(9 marks)

QUESTION FIVE (20 MARKS)

a) Given that $\vec{a} = (-1, 1, 3)$ and $\vec{b} = (7, -2, 2)$, find the angle between $\vec{a}$ and $\vec{b}$. (6 marks)

b) Given that the vectors $\vec{a} = (1, -6, 1)$ and $\vec{b} = (5, 2, \beta)$ are perpendicular, find the value of β. (4 marks)

c) Given that the vectors $\vec{a} = (1, 4, -2)$ and $\vec{b} = (2, -3, -1)$, find the cross product of $\vec{a}$ and $\vec{b}$. (4 marks)

d) Find the Eigen values and Eigen vectors of the matrix $A = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$. (6 marks)

END