TECHNICAL UNIVERSITY OF KENYA

FACULTY OF ENGINEERING SCIENCES AND TECHNOLOGY

SCHOOL OF INFRASTRATURE AND RESOURCE ENGINEERING

DEPARTMENT OF CIVIL AND CONSTRUCTION ENGINEERING

FIRST SEMESTER EXAMINATION FOR

BACHELOR OF TECHNOLOGY (CIVL) – YEAR 2

UNIT CODE: EICI 2101

UNIT NAME: MATHEMATICS IIA

DATE: SEP - DEC 2017 TIME: 2 HOURS

INSTRUCTIONS:

1) Answer Question ONE and any other TWO questions.
2) Mobile phones and any written material are prohibited in the examination room.
3) No writing should be done on this question paper. Any rough work should be done at the back of the answer booklet and canceled.
4) All answer booklets should be handed in at the end of the exam whether used or not.
5) Programmable calculators are prohibited.

QUESTION ONE (COMPULSORY) – 30MARKS

(a) Find the solution to the initial value problem

$y'' + 2y' + 2y = 0$, $y\left(\frac{\pi}{4}\right) = 2$, $y'\left(\frac{\pi}{4}\right) = -2$ (7marks)

(b) Find the limit of the function $f(x)$ as $x \to 1$ given,

$$f(x) = \begin{cases} 4 - 2x, & x < 1 \\ 4x - x^2, & x > 1 \end{cases}$$

(5marks)

(c) Apply Pythagoras theorem on the length of the hypotenuse of a right angled triangle if an infinitesimally small part of an arc length forms the hypotenuse to derive the formula for calculating the arc length given by

$$arc\ length = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

(5marks)

(d) Evaluate the arc length along $y = (x-1)^{\frac{3}{2}}$ from $x = 1$ to $x = 5$ to the nearest three decimal places. (7marks)

(e) Expand $(1 + x)e^x$ as a Taylor series in x

(6marks)

QUESTION TWO – 20MARKS

(a) i) Derive the formula for the summation of an arithmetic series given by

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

(5marks)

ii) The second term of an A.P is 6, and the eighth term is 30. Find the first term, the common difference, and the sum of the first 32 terms of the geometric progression

(7marks)

(b) Prove the theorem for the test for convergence of infinite series which states that If $\sum b_m$ converges then $\lim_{m\to\infty} b_m = 0$ (8marks)

QUESTION THREE – 20MARKS

(a) Evaluate $\int_0^{\frac{\pi}{6}} e^y \cos y\,dy$ correct to four decimal places (10marks)

(b) Determine the surface area of revolution the solid ~~of revolu~~tion obtained by rotating

$$y = \sqrt{4 - x^2} \qquad -4 \le x \le 4 \qquad \text{about the x – axis}$$

(10marks)

QUESTION FOUR – 20MARKS

(a) Use elementary row operation to solve the system of linear equations

$$x + 4y + z = 9$$
$$2x + 2y - 2z = 0$$
$$3x + y + 3z = 16$$

(8marks)

(b) Solve the system of linear equations below using the Lower – Upper Decomposition method.

$$z_1 + z_2 - z_3 = 4$$
$$z_1 - 2z_2 + 3z_3 = -6$$
$$2z_1 + 3z_2 + z_3 = 7$$

(12marks)

QUESTION FIVE – 20MARKS

(a) Solve the first order system $X' = \begin{pmatrix} X_1' \\ X_2' \end{pmatrix} = A_x = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$

(12marks)

(b) Determine if the following function is continuous at $x = 0$

$$f(t) = \begin{cases} \dfrac{x-6}{x-3}, & for\ x < 0 \\ 2 & for\ x = 0 \\ \sqrt{4 + x^2}, & for\ x > 0 \end{cases}$$

(8marks)