UNIVERSITY EXAMINATIONS 2016/2017
ORDINARY EXAMINATION FOR BSC ACTUARIAL SCIENCE
BAS2206 STOCHASTIC MODELS FOR ACTUARIES 1
(EVENING/ WEEKEND)

DATE: APRIL, 2017 **TIME: 2 HOURS**

INSTRUCTIONS: Answer question **ONE** and Any other **TWO** questions

QUESTION ONE (30 MARKS)

(a) Distinguish the following terms as used in stochastic Processes
   i. Stochastic Process and Stationary Process
   ii. Discrete time process and Continuous time process
   iii. States of a process and State space of a process [6 Marks]

(b) For the following transition matrix, draw a transition diagram to show the transition probabilities that the matrix describes [4 Marks]

$$P = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array} \begin{array}{c} \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \begin{pmatrix} 0 & p_{12} & 0 \\ 0 & p_{22} & p_{23} \\ p_{31} & 0 & p_{33} \end{pmatrix} \end{array}$$

(c) Suppose there are two markets products of brand A and Brand B respectively. Let each of these brands have exactly 50% of the total market share in the same period and let the market be of a fixed size. The transition matrix is given by the matrix:

$$\begin{array}{c} \\ From \end{array} \begin{array}{c} \begin{array}{cc} & To \end{array} \\ \begin{pmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{pmatrix} \end{array}$$

Determine the market share in the steady state [4 Marks]
How long will it take to reach the equilibrium state? [5 Marks]

(d) Define a Poisson Process, Stating its properties [5 Marks]

(e) Let X be a discrete random variable with probability generating function (PGF) $G_X(s) = \frac{5}{5}(3s^2 + s + 1)$.

i. Find the distribution of X [5 Marks]

ii. Hence or otherwise show that X is a pdf. [1 mark]

SECTION B

QUESTION TWO (20 MARKS)

(a) Define the following as used in stochastic processes

i. Periodicity [3 Marks]

ii. Ergodicity [3 Marks]

iii. Communicate [3 Marks]

(b) Let

$$P = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{array} \begin{array}{c} \begin{array}{ccccc} E_1 & E_2 & E_3 & E_4 & E_5 \end{array} \\ \begin{bmatrix} \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} \end{bmatrix} \end{array}$$

i. Draw a transition diagram to represent the transition matrix. [4 Marks]

ii. Re-arrange the matrix to be of the form

$$P = \begin{bmatrix} I & 0 \\ R & Q \end{bmatrix}$$

[3 Marks]

iii. Find P^2, hence P^n [4 Marks]

QUESTION THREE (20 MARKS)

(a) Discuss any three Practical importance of stochastic process. [8 Marks]

(b) A small town with one hospital has two ambulances to supply ambulance service. Requests for ambulances during non-holiday weekend averages 0.8 per hour and tend to be Poisson distributed. Travel and assistance time averages one hour per call and follows an exponential distribution.

i. What is the utilization of ambulances?

ii. On an average, how many requests are waiting for ambulances?

iii. How long will a request have to wait for ambulances?

iv. What is the probability that both ambulances are sitting idle at a given point in time? [12 Marks]

QUESTION FOUR (20 MARKS)

(a) On January 1 of this year Bakery A had 40% of its local market while Bakery B and C had 40% and 20 % of the market share respectively. Based on a study by a market Research firm, the following facts were compiled: Bakery A retains 90% of its customers gaining 5% from B and 10% from C. Bakery B retains 85% of its customers while gaining 5% from A and 7% from C. Bakery C retains 83% of its customers and gains 5% of A and 10% of B's customers

i. Formulate a transition matrix for the three operators
ii. What will each bakery share be on January of next year
iii. what will each bakery share be in the long-run [12 Marks]

(b) Discuss the Classification of stochastic processes giving examples. [8 Marks]

QUESTION FIVE (20 MARKS)

(a) Define the term random walk as used in stochastic processes. [3 Marks]

(b) List down the basic properties of a one dimensional wiener process. [6Marks]

(c) A company rates every employee as below average, average or above average. Past performance indicates that each year 10% of the below average employees will raise their rating to average and 25% of the average employees will raise their rating to above average. On the other hand, 15% of the average employees will lower their rating to below average and 15% of the above average employees will lower their rating to average. Company policy prohibits changes from below average to above average, or conversely in a single year.

(i) Come up with the transition matrix to represent this information. [4 Marks]
(ii) Calculate the percentage of employees who will receive below average rating over the long-run. [7 Marks]