

W1-2-60-1-6

**JOMO KENYATTA UNIVERSITY**

**OF**

**AGRICULTURE AND TECHNOLOGY**

**University Examinations 2015/2016**

**THIRD YEAR FIRST SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS/MATHEMATICS AND COMPUTER SCIENCE.**

**SMA 2301: REAL ANALYSIS I**

 **DATE AUGUST 2015 2HOURS**

**INSTRUCTIONS; ATTEMPT QUESTION ONE AND ANY OTHER TWO**

**QUESTION ONE (30 MARKS)**

1. Show that equivalence of sets is transitive. (3marks)
2. i) Differentiate between *countably infinite set* and *uncountable set*. (2marks)

ii) Prove that every subset of a countable infinite set is countable. (6marks)

1. i) Let X be a non-empty set. Explain what is meant by a metric on X. (2marks)

 ii)Suppose d is a metric on X. prove that d is neither one-to-one nor onto. (2marks)

1. i) Prove that if X and Y are sets of real numbers that are bounded below and, then. (3marks)

ii) Give examples of sets A and B with such that (1mark)

1. Suppose prove that A is closed if and only if. (4marks)
2. i) Using definition show that the sequence diverges. (5marks)

ii) Use part f) (i) above to conclude whether or not is Cauchy. (2marks)

**QUESTION TWO (20MARKS)**

1. Prove that the set of real numbers is uncountable. (6marks)
2. show that if a is a real number and is a positive real number, is an open ball centred at a and with radius (3marks)
3. Show that is not a neighbourhood of b. (6marks)
4. i) Find where possible the supremum and infimum of each of the sets below. (4marks)

And

 ii) Give an example of a set C of real numbers such that (1mark)

**QUESTION THREE (20MARKS)**

1. Use an example to show that

 i)A countable set need not to be bounded (1mark)

 ii) A bounded set need not be countable (1mark)

1. Show that if a real number x is a limit point of a set A of a real numbers, then x is not an isolated point of A. (4marks)
2. given that A and B are sets of real numbers show that (6marks)
3. prove that;
4. The union of an infinite collection of open sets is open. (4marks)
5. The union of an infinite collection of closed sets need to be closed. (2marks)
6. Use an appropriate counter example to prove that if A is a neighbourhood of x and, then B is not necessarily a neighbourhood of x. (2marks)

**QUESTION FOUR (2MARKS)**

1. Given reasons, state whether each of the following statements is true or false.
2. The isolated point of are the integers themselves. (2marks)
3. The boundary points of are all integers. (2marks)
4. Consider the set Determine
5. The interior of A. (3marks)
6. The closure of A. (4marks)

Hence conclude that A is neither open nor closed. (2marks)

1. i) Explain what is meant by ‘a set of real numbers is nowhere dense in (2marks)

 ii) Give an example of a subset of that is nowhere dense in. (1mark)

1. Prove that if a set has a supremum, then then supremum is unique. (4marks)