UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (COMPUTER SCIENCE) AND BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

CSC 125/SIT 125: LINEAR ALGEBRA

DATE: APRIL 10, 2018 **TIME: 8:30 AM – 10:30 AM**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) Let $\boldsymbol{u} = (-4,7)$, $\boldsymbol{v} = (5,2)$ be two vectors in $\mathbb{R}^2$. Determine $|3u - 7v|$. (2 marks)

b) Given that $\boldsymbol{u} = \begin{bmatrix} lnx \\ x^2 \\ sinx \end{bmatrix}$ determine $\frac{d^3u}{dx^3}$ (3 marks)

c) Let $\boldsymbol{v}$ be a vector space over $\mathbb{R}$. Explain what is meant by a bilinear form on $\boldsymbol{v}$. (3 marks)

d) Given the vectors $\boldsymbol{u} = (k^2, -1,3), \boldsymbol{v} = (2, k, -5)$, find k such that $\boldsymbol{u}$ and $\boldsymbol{v}$ are orthogonal. (2 marks)

e) Solve the system of linear equations using the inverse matrix method

$$2x + y = 7$$

$$3x - 2y = 0$$

(4 marks)

f) If $A = \begin{bmatrix} t^2 & \frac{5}{t} \\ e^t & 3t \end{bmatrix}$ determine $\int Adt$. (2 marks)

g) Find the values of k for which the matrix $\begin{pmatrix} 3-k & 6 \\ 2 & 4-k \end{pmatrix}$ is singular. (3 marks)

h) Given vector $\boldsymbol{a} = 3i + 2j, \boldsymbol{b} = 2i + kj$ where k is a scalar, find k such that $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel. (2 marks)

i) Describe an eigenvalue as used in linear algebra (2 marks)

j) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by $T(x, y) = xy$. Determine whether T is a linear transformation. (2 marks)

k) Find an L U decomposition of $\begin{bmatrix} 2 & 1 \\ -4 & -6 \end{bmatrix}$ hence solve the system of linear equation

$$2x_1 + x_2 = 1$$
$$-4x_1 - 6x_2 = 2$$

(5 marks)

QUESTION TWO (20 MARKS)

a) Determine the vector product $\boldsymbol{u} \times \boldsymbol{v}$ given that $\boldsymbol{u} = (2, -1,3), \boldsymbol{v} = (-1,3,1)$ (4 marks)

b) Find the inverse of the following matrix by first getting the adjoint matrix

$$\begin{pmatrix} -7 & 5 & 3 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \end{pmatrix}.$$

Hence or otherwise solve the system of linear equations

$$-7x_1 + 5x_2 + 3x_3 = 6$$
$$3x_1 - 2x_2 - 2x_3 = -3$$
$$3x_1 - 2x_2 - x_3 = 2$$

(11 marks)

c) If $A = \begin{bmatrix} 3 & 5 \\ 2 & -3 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & 5 \\ -2 & 0 \end{bmatrix}$ find $A^T A + 3B$. (5 marks)

QUESTION THREE (20 MARKS)

a) Let $\boldsymbol{u} = (1, -2, 3), \boldsymbol{v} = (-2, -3, 2)$ be two vectors in$\mathbb{R}^3$. Determine:

i) The vector projection of **u** onto ***v***. (3 marks)

ii) The scalar projection of ***u*** onto ***v***. (2 marks)

b) Compute the eigenvalues and the corresponding eigenvectors of $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

(11 marks)

c) Determine the symmetric matrix A corresponding to the quadratic form

$$Q(x) = Q(x_1, x_2, x_3) = 3x_1^2 + 4x_1x_2 - x_2^2 + 8x_1x_3 - 6x_2x_3 + x_3^3$$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Let $\boldsymbol{u} = (1, -3, 4), \boldsymbol{v} = (3, 4, 7)$ be two vectors in$\mathbb{R}^3$. Find:

i) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$. (3 marks)

ii) A unit vector in the direction of $\boldsymbol{u}$. (2 marks)

iii) The distance between $\boldsymbol{u}$ and $\boldsymbol{v}$. (3 marks)

b) Determine whether the function $T: \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(x, y, z) = (x - y, y - z, x - z)$ is a linear transformation. (6 marks)

c) Use Cramers rule to solve the system of linear equations

$$2x + y - 3z = 9$$
$$x - y + 2z = -2$$
$$3x + 2y + 2z = 8$$

(6 marks)

QUESTION FIVE (20 MARKS)

a) Consider the function $B: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ defined as follows $B(x, y) = x_1y_2 + x_2y_1$ for all $x = (x_1, x_2)$ $y = (y_1, y_2) \in \mathbb{R}^2$. Determine if B is a bilinear form on $\mathbb{R}^2$ (7 marks)

b) Using Gaussian elimination method, determine the inverse of the matrix

$$B = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}$$

(6 marks)

c) Let $\boldsymbol{v_1} = (2, 3, 1)$, $\boldsymbol{v_2} = (1, -1, 2)$ $\boldsymbol{v_3} = (7, 3, 8)$, be three vectors in $\mathbb{R}^3$. Determine if $\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}$ are linearly independent or not. (7 marks)

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