UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF EDUCATION SCIENCE & BACHELOR OF EDUCATION ARTS

SMA 306: COMPLEX ANALYSIS 1

DATE: APRIL 7, 2017 **TIME: 8:30-10:30 AM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 MARKS)

a) Given that $z_1 = 3 + i$ and $z_2 = 5 - i$ rationalise the denominator of the complex fraction $\frac{z_1}{z_2}$ giving your answer in the simplest form. (3 Marks)

b) Express $z = -\sqrt{6} - \sqrt{2i}$ in polar form. (4 Marks)

c) Prove that $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$ (4 Marks)

d) Find all the points of discontinuity of the function $F(Z) = \frac{Z^2+1}{Z^4-16}$ (5 Marks)

e) Show that $F(Z) = e^z$ is a periodic function with the period $2k\pi i$ (3 Marks)

f) Evaluate $\int \overline{Z}\, dz$ from Z=0 to Z = 4+2i along the line joining Z = 0 to Z = 2i and the line joining Z = 2i to Z = 4+2i. (5 Marks)

g) Evaluate

$$\oint_c \frac{e^z}{(z-2)(z-4)} dz$$

When $|z| = 5$ (4 Marks)

h) State without proof the Cauchy's fundamental theorem. (2 Marks)

QUESTION TWO (20 MARKS)

a) Prove that

i. $U = e^{-x}(x\sin y - y\cos y)$ is harmonic. (6 Marks)

ii. Find V such that $F(z) = U + iV$ is analytic. (U is given in (i) above) (5 Marks)

b) Show that $|e^z| = e^x$ (3 Marks)

c) Prove that if F(Z) is analytic inside and on the boundary C of a simply connected region R and a is any point inside the curve C then $\oint_C \frac{F(Z)}{Z-a} dz = 2\pi i F(a)$. (6 Marks)

QUESTION THREE (20 MARKS)

a) Find the Taylor's series of expansion f(z) = $\frac{1}{(z+1)^2}$ about z= -i (4 Marks)

b) Show that $Sin^{-1} z = \frac{1}{i} ln(iz + \sqrt{1-z^2})$ (7 Marks)

c) Evaluate $\int_{c_1} z^2 dz$ where c_1 is the line segment OB from $z = 0$ to $z = 2 + i$ (3 Marks)

d) Given that $f(z) = \frac{1}{z(z+2)}$ resolve into partial fractions and hence use Cauchy's theorem to evaluate $\oint_c \frac{dz}{z(z+2)}$ (6 Marks)

QUESTION FOUR (20 MARKS)

a) Find the poles and the corresponding residues of each of the following functions.

(i) $F(z) = \frac{z^3+5z+1}{z-2}$ (3 Marks)

(ii) $f(z) = \frac{z}{(z-1)(z+1)^2}$ (6 Marks)

b) Use the residue theorem to evaluate $\oint_c \frac{e^z}{(z-2)(z-4)} dz$ when $|z| = 5$ (6Marks)

c) Expand f(z) = $\frac{3}{z^2(z-3)^2}$ in a Laurent series at z=3. (5 Marks)

QUESTION FIVE (20 MARKS)

a) Use the Cauchy Integral formula to evaluate $\oint_C \frac{\cos Z}{Z^3}\, dz$ (3 Marks)

b) Use De Moivre's theorem to prove the identity

$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (6 Marks)

c) Evaluate $\lim_{z\to i} \frac{z^2+1}{z^6+1}$ (3Marks)

d) Show that $F(z) = e^{-y}(\cos x + i\sin x)$ is analytic. (4 Marks)

e) Given the Cauchy Riemann's equations

$\frac{du}{dx} = \frac{dv}{dy}$ *and* $\frac{dv}{dx} = -\frac{du}{dy}$ derive the Laplace's equations if the second derivative of U and V exists and are continuous. (4 Marks)

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