MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE WITH IT

SAC 208: RISK THEORY

INSTRUCTIONS: Attempt Question ONE and ANY OTHER TWO questions

QUESTION 1 [COMPULSORY] [30 Marks]

(a) Define the following

(i) The moment generating function of a random variable X [**3 Marks**]

(ii) Risk premium [**3 Marks**]

(iii)Excess of loss Reinsurance [**3 Marks**]

(b) The individual loss amounts against a portfolio are Uniformly distributed on [0,m]. For 75% of the losses $m = 1000$. For another 25% of the losses we have $m = 2000$. Write down a formular for the p.d.f of a randomly selected loss from this portfolio. [**3 Marks**]

(c)Suppose that a ground-up loss X is Uniformly distributed on the interval [0,1000]. Suppose there is a policy limit of 500 per loss.

Determine the insurer's loss payment per loss. [**5 Marks**]

(d)A group of 15 policyholders each have the same probability of making a claim on their policies. If the probability of making a claim is 0.15, what is the probability that at least one policyholder out of the group makes a claim? Assume modelling by a Binomial distribution. [**5 Marks**]

(e)The number of claims, N, made on an insurance portfolio follows the following distribution

n	$Pr(N = n)$
0	0.7
2	0.2
3	0.1

If a claim occurs, the benefit is 0 or 10 with probability 0.8 and 0.2 respectively. The number of claims and the benefits for each claim are independent.

Calculate the probability that the aggregate benefits will exceed expected benefits by more than 2 standard deviations. **[8 Marks]**

QUESTION 2 [20 Marks]

The annual frequency from a portfolio,N, follows a Poisson distribution with mean $\lambda = 5$

$$Pr(N = n) = \frac{e^{-5}5^k}{k!}$$

The individual loss amounts Y, follow an exponential distribution with pdf

$$f_Y(y) = 0.002e^{-0.002y}, \qquad y > 0$$

Let S be the aggregate annual loss for the portfolio. Determine

$(a)E(S)$ **[3 Marks]**

$(b)Var(S)$ **[3 Marks]**

$(c)M_s(t)$ **[3 Marks]**

$(d)Pr(S = 0)$ **[3 Marks]**

QUESTION 3 [20 Marks]

(a)A towing company provides all towing services to members of the City Automobile Club. You are given

Towing distance (Km)	Towing cost(Kshs)	Frequency
0-9.99	80	50%
10-29.99	100	40%
30+	160	10%

(i)The automobile owner must pay 10% of the cost and the remainder is paid by the City Auto Club

(ii)The number of towings has a Poisson distribution with mean 1000 per year

(iii)The number of towings and the costs of individual towings are mutually independent

Using normal approximation for the distibution of the aggregate towing costs, calculate the probability that the City Auto Club pays more than Kshs.90,000 in any given year. [**10 Marks**]

(b)The number of auto vandalism claims per month at Sunny Daze Insurance Company (SDIC) has mean 110 and variance 750. Individual losses have mean 1101 and standard deviation 70. The number of claims and the amount of individual losses are independent.

Using normal approximation, calculate that SDIC's aggregate auto vandalism losses reported will be less that 100,000. [**10 Marks**]

QUESTION 4 [20 Marks]

(a)For an insurance portfolio

(i)The number of claims has a Binomial distribution with $n = 5$ and $p = 0.05$

(ii)Each claim amount has a Poisson distribution with mean 10

(iii)The number of claims and the claim amount are mutually independent

Calculate the mean and the variance of the aggregate claims [**10 Marks**]

(b)Two types of insurance claims are made to an insurance company. For each type, the number of claims follow a Poisson distribution and the amount of each claim is Uniformly distributed as follows

Type of claim	Poisson parameter λ for no. of claims	Range of claim amount
I	12	(0,1)
II	4	(0,5)

The number of claims of the two types are independent and the claim amounts and claim numbers are independent. Calculate the normal approximation to the probability that the total claim amounts exceed 18.[**10 Marks**]

QUESTION 5[20 Marks]

Suppose an insurer has 100 independent policies. Assume that over the next year each policy will generate either 0 or 1 claim with respective probabilities 0.90 and 0.10. Assume that each claim amount is uniformly distributed on (0,1000].

$Pr(N = 0) = 0.9$

$pr(N = 1) = 0.10$

$f_Y(y) = 0.001$ for $0 < y < 1000$

Supppose we let S denote the insurer's aggregate claims in the next year. Determine the mean and the variance of the aggregate claim amounts assuming:-

(a)An Individual Risk Model [**10 Marks**]

(b)A Collective Risk model [**10 Marks**]

Comment on your answers above.