KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2013/2014

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 431: DIFFERENTIAL GEOMETRY

DATE: FRIDAY 4TH APRIL 2014 **TIME:** 11.00 A.M. – 1.00 P.M.

INSTRUCTIONS

Answer question ONE and any TWO of the remaining questions

Question 1

a) i) Find a unit vector perpendicular to both

$\overline{A} = \overline{i} + 2\vec{k}$ *and* $\overline{B} = 2\vec{k} - 2j$ (3 marks)

ii) Find the volume of the parallelepiped formed by the vectors

$\vec{a} = \overline{i} - \bar{j} + \overline{k}$, $\vec{b} = 3i + j - 2\vec{k}$, $\vec{c} = -i + 2\vec{j} - 3\vec{k}$

4marks

b) Write the vector and symmetric equations of the tangent line to the curve

$\vec{r} = \{t^2 - 2 , t + 2 , t^4 + 4t + 1\}$ (4 marks)

at a point corresponding to $t = 1$

c) i) Prove that a regular curve $\vec{r} = \overline{r}(s)$ of class ≥ 2 is a straight line if and only if the curvature is zero. (3 marks)

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ii) Explain briefly what you understand by the moving trihedron on a regular curve

$\bar{r} = \bar{r}(s)$ where s is arc-length (3 marks)

d) i) Give the definition of the First Fundamental form on a regular surface $\vec{r} = \vec{r}\,(u, v)$. (4 marks)

ii) Obtain the First Fundamental form on a regular surface given by

$x = u\cos v \;,\; y = u\sin v \;,\; z = u^2$ (5 marks)

Where u and v are independent parameters

e) Find the length of the arc of the curve $x = ct \;,\; y = c\sqrt{2}\,\ell nt \;,\; z = \frac{c}{t}$

between the points corresponding to $t = 1$, and $t = 10$ (4 marks)

Question 2 (20 marks)

a) Obtain the arc-length of the section of the curve

$\vec{r} = \{\cos^3 t, \sin^3 t, \cos 2t\}$ for $0 \le t \le \pi/3$ (5 marks)

b) i) Show that the curvature $|K|$ of a curve $\bar{r} = \bar{r}(s)$ of class ≥ 2, is equal to

$\lim_{\Delta s \to 0} \frac{\Delta\theta}{|\Delta s|} = \frac{d\theta}{ds}$, where $\Delta\theta$ – angle

between the unit tangent vectors

$\vec{T}(s)$ at $\bar{r}(s)$ and $\bar{T}(s+\Delta s)$ at $\bar{r}(s+\Delta s)$

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ii) Determine the intrinsic equations of the hyperbolic helix

1 mark

$\vec{r} = \{2cht\ ,\ 2sht\ ,\ 2t\}$

Question 3 (20 marks)

a) Write briefly on the angle between directions $\delta\vec{r}$ and $d\vec{r}$ of two lines on a regular surface $\bar{r} = \bar{r}\ (u\,v)$ at a point $(u\ ,v)$ and show that the angle can be expressed in terms of the coefficient of the First Fundamental form on the surface.

(8 marks)

b) Using the above definition, determine the angle between the lines $v = 2u$ and $v = -2u$ on a surface having the First Fundamental form $ds^2 = du^2 + dv^2$

(12 marks)

Question 4 (20 marks)

a) i) Write the equations of the tangent line and the normal plane to the helix $x = a\cos t,\quad y = b\sin t\ ,\quad z = e^t$ at a point where $t = 0$ (6 marks)

ii) Show that the area of the Monge's Patch $\vec{r} = \{u\ ,\ v\ ,f\ (u,v)\}$ is given as

$$A = \iint_W \sqrt{1 + f_u^2 + f_v^2}\ du\,dv$$ (6 marks)

b) Explain (with the help of a diagram) what you understand by normal curvature to a curve $\bar{r} = \bar{r}\ (s)$ on the surface $\vec{r} = \bar{r}\ (u,\ v)$ and obtain its expression in terms of the coefficients of First and Second Fundamental forms. (8 marks)

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Question 5 (30 marks)

a) Obtain the unit normal and equation of the tangent plane to the surface

$\overline{\gamma} = \{u + v\ ,\ u - v, uv\}$ at a point where $u = 2\ ,\ v = 1$ (6 marks)

b) Find the equations for the

i) Osculating plane

ii) Normal plane (6 marks)

iii) Rectifying plane to the curve

$x = 3 - t^3\ ,\ y = 3t^2\ ,\ \ z = 3t + t^3$ at the point where $t = 1$

c) Obtain the Second Fundamental form on the surface $\overline{r} = \{u\cos v\ ,\ u\sin v\ ,\ hv\}$

(8 marks)

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