MUEO

MOI UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR
(ACADEMICS, RESEARCH & EXTENSION)

UNIVERSITY EXAMINATIONS

2018/2019 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER EXAMINATION

FOR THE DEGREE OF

BACHELOR OF EDUCATION SCIENCE

COURSE CODE: PHY 310/314

COURSE TITLE: QUANTUM MECHANICS I

DATE: 10TH JULY, 2019 **TIME:** 9.00 A.M. – 12.00 NOON

INSTRUCTION TO CANDIDATES

- SEE INSIDE

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UNIVERSITY EXAMINATIONS
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THIRDYEAR, SEMESTER II EXAMINATION

FOR THE DEGREES OF
BACHELOR OF EDUCATION SCIENCE

COURSE CODE: PHY 310/314

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INSTRUCTIONS TO CANDIDATES

- **Answer question ONE, TWO and any otherTHREE questions. Section A carries 28 marks and all the other questions in section B carry 14 marks each.**
- **Illustrate your answers with suitable diagrams wherever necessary.**
- **Duration of the examination: 3 hours**

You may find the following table useful.

Observable	symbol	Associated Operator
Position	x	x
momentum	p	$\frac{\hbar}{i}\frac{\partial}{\partial x}$
Potential energy	U	$U(x)$
Kinetic energy	K	$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
Hamiltonian	H	$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)$
Total energy	E	$i\hbar\frac{\partial}{\partial t}$

$h = 6.63\times10^{-34}\, J.s$

$m_e = 9.1\times10^{-31}\, kg$

$1eV = 1.6\times10^{-19}\, J$

$c = 3.0\times10^{8}\, m/s$

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SECTION A

Question 1

(i) The complex conjugate of $z = a + bi$ is $z = a - bi$denoted by$\bar{z}$. Show that (a) $z\bar{z} = |z|^2$; (b) $z + \bar{z}$ is reall. [2 marks]

(ii) Prove that for the operators A,B and C, the following identities are valid:

(a) [B,A]=-[A,B] [1 mark]

(b) [A+B,C]=[A,C]+[B,C] [2 marks]

(c) [A,BC]=[A,B]C+B[A,C] [2 marks]

(iii) Suppose the operators A and B commute with their commutator, i.e [B,[A,B]]=[A,[A,B]]=0. Show that

(a) $[A, B^2] = nB^{n-1}[A, B]$; [2 marks]

(b) $[A^n, B] = nA^{n-1}[A, B]$. [1 marks]

(iv) Prove that the Bohr hydrogen atom approaches classical condition when n becomes very large and small quantum jumps are involved. [5 marks]

(v) (a) Consider a thermal neutron, that is, a neutron with speed v corresponding to the average thermal energy at the temperature T=300k. Is it possible to observe a diffraction pattern when a beam of such neutron falls on a crystal? [3 marks]

(b) In a large accelerator, an electron can be provided with energy over $1GeV = 10^9 eV$. What is the De Broglie wavelength corresponding to such electron?[2 marks]

Question Two

Consider a particle of mass m and energy $E > 0$ held in the one-dimensional potential $-V_0\delta(x - a)$.

(a) integrate the stationary Schrodinger equation between $a - \varepsilon$ and $a + \varepsilon$. Taking the limit $\varepsilon \to 0$, show that the derivatives of the eigen function $\emptyset(x)$ presents a discontinuity at $x = 0$ and determine it. [3 marks]

(b) from part (a) $\emptyset(x)$ can be written as

$$\begin{cases} \emptyset(x) = A_1 e^{ikx} + A_1' e^{-ikx} & x < a \\ \emptyset(x) = A_2 e^{ikx} + A_2' e^{-ikx} & x > a \end{cases}$$

Where $k = \sqrt{2mE/\hbar^2}$. Calculate the matrix M defined by $\begin{pmatrix} A_2 \\ A_2' \end{pmatrix} = M \begin{pmatrix} A_1 \\ A_1' \end{pmatrix}$ [5 marks]

Section B

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Question Three

Derive the following properties of the ad joint of an operator: (a) $(A^\dagger)^\dagger = A$; [3 marks]

(b) $(\lambda A)^\dagger = \lambda^* A^\dagger$, where λ is a complex number; [4 marks]

(c) $(A+B)^\dagger = A^\dagger + B^\dagger$; [3 marks]

(d) $(AB)^\dagger = B^\dagger A^\dagger$ [4 marks]

Question Four

(i) Consider a one-dimensional oscillator in the nth energy level. Compute the expectation values$\langle x^2\rangle, \langle X^2\rangle, \langle p^2\rangle, \langle P^2\rangle$.

What can you say about the uncertainty relation $\Delta x \Delta p$? [5 marks]

(ii) Prove the following relations for the angular momentum operator:

(a) $[L^2, L] = 0$; [6 marks]

(b) $L \times L = i\hbar L$ [3 marks]

Question Five

Using the basis vectors of S_z eigenvectors, calculate$S_i\left|+\frac{1}{2}\right\rangle$and $S_i\left|-\frac{1}{2}\right\rangle$ $(i = x, y, z)$, where $\left|+\frac{1}{2}\right\rangle$ and $\left|-\frac{1}{2}\right\rangle$ are the eigenvectors of $S_z$with eigenvalues ${}^{+\hbar}/_2$ and ${}^{-\hbar}/_2$, respectively. The basis vectors of S_z eigenvectors are $S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ *and* $S = \frac{\hbar}{2}\sigma$ [7 marks]

(b) calculate the commutation relations $[\sigma_i, \sigma_j]$, where $j = x, y, z$ and σ_i are the Pauli matrices, i.e

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

$E = \sqrt{p^2c^2 + M_e^2c^4}$

[7 marks]

Question Seven

Consider a particle in a central potential. Given that $|lm\rangle$ is an eigenstate of L^2andL_z: (a) compute the; sum$\Delta L_x^2 + \Delta L_y^2$. [12 marks]

(b) For which values of l and m does the sum in part (a) vanish? [2 marks]

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