CHUKA UNIVERSITY

RESIT/ SPECIAL EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF BACHELOR OF

MATH 400: TOPOLOGY 1

STREAMS: **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 12/09/2018 **2.30 PM – 4.30 PM**

INSTRUCTIONS:

- Answer **ALL** questions
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a) Consider the following topology on $X=\{a,b,c,d,e\}$ and

$\tau=\{\{a\},\{a,b\},\{a,c,d\},\{a,b,e\},\{a,b,c,d\},X,\varnothing\}$.

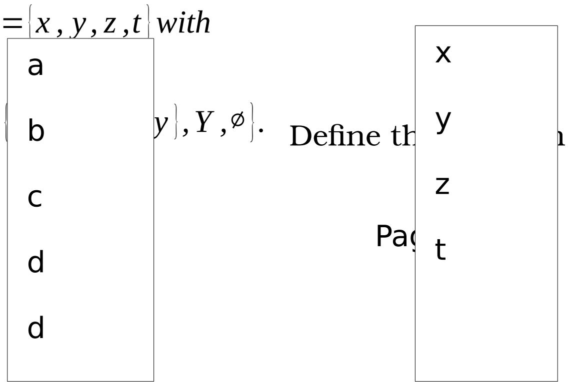
Given the sets $A=\{a\},B=\{b\},C=\{c,e\}$, which ones are dense in X?

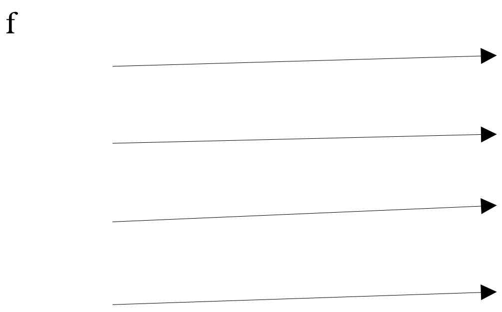
(7mks)

(b) Let $X=\{a,b,c,d\}\ with\ \tau_X=\{\{a,b\},\{a\},\{b\},X,\varnothing\}$ and Let

$Y=\{x,y,z,t\}\ with$

$\tau_Y=\{$ … $y\},Y,\varnothing\}$. Define th… f as





Show that the function f is a homomorphism. (5mks)

(c) Let $f: X \to Y$ be a constant function. Prove that then f is continuous relative to τ_X and τ_Y.

(4mks)

(d) (i) Let $X = \{a, b, c, d, e\}$. Determine whether or not the class $\tau_A = \{\{a, b, c\}, \{a, b, c, d\}, \{a, b, d\}, X, \varnothing\}$ of subsets of X is a topology on X.

(4mks)

(ii) Let τ be a topology on a set X consisting of four sets i.e. $\tau = \{X, \varnothing, A, B\}$. What conditions must A and B satisfy?

(3mks)

(e) Prove that if A is a subset of a discrete topology, then set of its derived points A' is empty

(6mks)

QUESTION TWO: (20 MARKS)

(a) Consider the following topology on $X = \{a, b, c, d, e\}$ and $\tau = \{\{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}, X, \varnothing\}$. If $A = \{a, b, c\}$. Find

(i) The exterior of A (3mks)

(ii) The boundary of A

(2mks)

(iii) Hence show that the boundary of A, $\delta A = \acute{A} \cap X\acute{/}A$ (3mks)

(b) Let $A \subset X$, where X is a non-empty topological space. Prove that $\acute{A} = \delta A\, A^0$

(7mks)

(c) Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be continuous functions. Prove that the composite function $g \circ f$ is continuous

(5mks)

QUESTION THREE: (20 MARKS)

(a) Let $p \in X$ and denote N_P the set of all neighborhood of a point p . Prove that

(i) $N_P \neq \emptyset \forall N \in N_P, p \in N$

(ii) $\forall\, pairs\, N, M \in N_P, N \cap M \in N_P$

(iii) If $N \in N_P$ and for every $M \subset X$ with $N \subset M$ it implies that $M \in N_P$

(6mks)

(b) Let $f: x_1 \rightarrow x_2 where\, x_1 = x_2 = \{0,1\}$ and are such that $(x_1, D) \wedge (x_2, \$)$ be defined by $f(1) = 1\, and f(0) = 0.$ Show that f is continuous whereas f^{-1} is not.

(6mks)

(c) Distinguish the following terms as used in topology

(i) An indiscrete topology and Sierpinski topology

(ii) A base for the topology τ and a local basis at the point p

(iii) A $T_1 \wedge T_2$ space

(iv) A regular space and a normal space (8mks)
