

CHUKA



UNIVERSITY

UNIVERSITY SUPPLEMENTARY/SPECIAL EXAMINATIONS.

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF

MATH 121/111: CALCULUS I

STREAMS:

TIME: 2 HOURS

DAY/DATE: MONDAY 23/07/2018

5.00 P.M - 7.00 P.M

INSTRUCTIONS:

- All Questions are COMPULSORY
- Do not write anything on the Question paper.

QUESTION ONE. [30 MARKS]

(a) Given $x = \frac{t}{1-t} \wedge y = \frac{t^3}{1-t}$, determine $\frac{dy}{dx}$ at $x=1$ [3 Marks]

(b) Evaluate $\int_0^1 2x - 3x^2 + 4x^3 dx$ [3 Marks]

(c) Evaluate $\lim_{x \rightarrow \infty} \sqrt[3]{\frac{x^2 + 9x^{-6}}{13 + 125x^2}}$ [3 Marks]

(d) Using first principles, determine the derivative of $y = \frac{7}{x}$ [4 Marks]

(e) Determine the equation of the tangent to the curve $y = \frac{3}{x^2}$ at $x=1$ [4 Marks]

(f) Using $y = \sqrt{x}$, determine $\sqrt{63}$ without a calculator or mathematical tables. [3 Marks]

(g) Find the areas enclosed by $y = 2 - x^2$ and $y = x^2 - 4x + 2$. [5 Marks]

(h) Using the first derivative test, find the interval for which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing. [5 Marks]

QUESTION TWO [20 MARKS]

(a) Find the derivative $\frac{dy}{dx}$ of the following functions:

(i) $y = \frac{x-5}{x+5}$ [3 Marks]

(ii) $y = 4e^{-3x^2}$ [3 Marks]

(iii) $y = (2x+1)(5x^2-7)$ [2 Marks]

(iv) $\frac{y}{x} = e^y$ [3 Marks]

(v) $y = x^{x^2}$ [3 Marks]

(vi) $xy^3 + y^2 - 5xy - x^2 = 7$ [3 Marks]

(b) If $x = \sin t$, $y = \cos 2t$, show that $\frac{d^2y}{dx^2} + 4 = 0$ [3 Marks]

QUESTION THREE [20 MARKS]

(a) A spherical balloon is blown so that its volume increases at the rate of $0.5 \text{ cm}^3/\text{s}$. Determine the rate at which the radius increases when the volume of the balloon is 32 cm^3 .

[6 Marks]

(b) Sketch the graph of $f(x) = \frac{2(x^2-9)}{x^2-4}$ [7 Marks]

(c) Find the co-ordinates of the point on the curve $y = x^3 - 6x^2 + 12x + 2$ at which the tangent is parallel to the line $y - 3x = 0$. [7 Marks]

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