CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS, BACHELOR OF SCIENCE, BACHELOR OF ARTS (MATHS/ECONS), BACHELOR OF SCIENCE (ECON STATS)

MATH 201: LINEAR ALGEBRA I

STREAMS: BED (SCI & ARTS), BSC, BA (MATHS-ECON), BSC (ECON STAT)(Y2S2)
TIME: 2 HOURS

DAY/DATE: WEDNESDAY 11/4/2018 11.30 A.M. – 1.30 P.M.

INSTRUCTIONS:

- **Answer question ONE and TWO other questions**
- **Sketch maps and diagrams may be used whenever they help to illustrate your answer**
- **Do not write anything on the question paper**
- **This is a closed book exam, no reference materials are allowed in the examination room**
- **There will be No use of mobile phones or any other unauthorized materials**
- **Write your answer legibly and use your time wisely**

QUESTION ONE (30 MARKS)

(a) Consider the system in unknown x and y

$x + ay = 4$

$ax + 9y = b$

Find which values of a does the system have a unique solution, and for which pairs of values (a, b) does the system have more than one solution.

(b) Evaluate the WROSKIAN $W(e^{x}, e^{-x}, e^{-2x}, 0)$ [4 marks]

(c) Distinguish the Kernel and range of a transformation T. Hence prove that if $T: U \rightarrow V$ is linear transformation, then the kernel of T is a subspace of U. [5 marks]

(d) Show that the subset $W = \{(x, y): x \geq 0, y \geq 0, x, y \in R^2\}$ is not a subspace of R^2

[3 marks]

(e) For any vector $v = (v_1, v_2)$in R^2, define T: $R^2 \to R^3$ defined by

$T(v_1, v_2) = \left(v_1 - v_{2,} 3v_1 - \ 2v_2, v_1, 2v_2\right)$, show that is a linear transformation.[5 marks]

(f) Determine if $p_1 = 1 - t, p_2 = 2 - t + t^2$ and $p_3 = 2t + 3t^2$ is a basis for the vector space $p_2(t)$ of polynomials of degree less or equal to 2. [5 marks]

(g) Given the following basis for $\mathbb{R}^3 B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $B^{'} = \{(1, 0, 1), (2, 1, 2), (1, 2, 2)\}$ find a transition matrix from B to B' [4 marks]

QUESTION TWO (20 MARKS)

(a) Using the concept of elementary product show that the determinant of the given matrix is the product of elements of the leading diagonals.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}$$

[4 marks]

(b) Let $A = \begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is the equation $\boldsymbol{Ax} = \boldsymbol{b}$ consistent for all values of $b_1 b_2 b_3$? Verify [4 marks]

(c) By use of the concept of rank of matrix, determine the type of solution to the following system of equations

$2x_1 + x_2 + x_3 = 1$

$-x_1 + 2x_2 + 3x_3 = 3$

$x_1 + 3x_2 - 2x_3 = 4$ [6 marks]

(d) For which values of a and b does the below system has

(i) No solution

(ii) Unique solution

(iii) Infinitely many solutions

$x_1 - 2x_2 + 3x_3 = 4$

$2x_1 - 3x_2 + ax_3 = 5$

$3x_1 - 4x_2 + 5x_3 = b$ [6 marks]

QUESTION THREE (20 MARKS)

(a) Using of row reduction method, find the inverse for the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & -3 \\ 2 & 1 & 5 \end{bmatrix}$ hence solve the system

$x + y + 2z = 2$

$x + y - 3z = 2$

$2x + y + 5z = 5$ [8 marks]

(b) Let $F: \mathbb{R}^4 \to \mathbb{R}^3$ be linear mapping defined by $F(x, y, z, t) = (x - y + z + t, 2x - 2y + 3z + 4t, 3x - 3y + 4z + 5t)$. Find

(i) The basis and dimension of the kernel of F

(ii) A basis and dimension of the image of F

(iii) Using the parts (i) and (ii) above, verify the dimension theorem [7 marks]

(c) Prove that if $S = \{v_1, v_2, \dots, v_n\}$ is a basis for a vector space V, then every set containing more than n vectors is linearly dependent. [5 marks]

QUESTION FOUR (20 MARKS)

(a) Let $T: \mathbb{R}^4 \to \mathbb{R}^2$ be a linear transformation defined by $T(x, y) = (2x - 2y, -x + 3y)$

(i) Find the matrix of T relative to the basis $B = \{(1, 0), (1,1)\}$ [3 marks]

(ii) Find the matrix of T relative to the basis $B' = \{(1, -1), (1,2)\}$ [3 marks]

(iii) Find the transition matrix P from the basis B to the basis B' and verify the relation $P^{-1}[T]_B P = (T)_{B'}$ [3 marks]

(b) Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}, u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}, c = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$ and define a transformation

T: $\mathbb{R}^2 \to \mathbb{R}^3$ by $T(x) = Ax$ so that $T(x) = Ax = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 5x_2 \\ -x_1 + 7x_2 \end{bmatrix}$

(i) Find $T(\boldsymbol{u})$, the image of $\boldsymbol{u}$ under T [1 mark]

(ii) Find an $\boldsymbol{x}$ in whose image under Tis $\boldsymbol{b}$ [5 marks]

(iii) Determine if $\boldsymbol{c}$ the range of the transformation T is [5 marks]

QUESTION FIVE (20 MARKS)

(a) Use Cramer's method to solve the system of equation

$x + y - 2z = -3$

$w + 2x - y = 2$

$2w + 4x + y - 3z = -2$

$w - 2x - 7y - z = 5$ [8 marks]

(b) Find the basis and dimension of the solution space for the equations

$2x_1 + 2x_2 - x_3 + x_5 = 0$

$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$

$x_1 + x_2 - 2x_3 - x_5 = 0$

$x_3 + x_4 + x_5 = 0$ [6 marks]

(c) For a matrix $A_{(m\ x\ n)}$, prove that

(i) If A is invertible, then Ax=b has a unique solution for any b [4 marks]

(ii) If A is row equivalent to an identify matrix l_n, then A is invertible.[4 marks]
