CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF ARTS AND BACHELOR OF EDUCATION

MATH 242: PROBABILITY AND STATISTICS II

STREAMS: BSC, BED & BA **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 18/04/2018 **11.30 A.M. – 1.30 P.M.**

INSTRUCTIONS:

- **Answer question ONE and any other TWO questions.**

QUESTION ONE (30 MARKS)

(a) State the central limit theorem. (2 marks)

(b) Let $\acute{X}$ be the mean of a random sample of size 16 from a normal distribution with mean $\mu = 800$ and variance $\sigma^2 = 1600$. Determine the value k such that $P(\acute{X} < k) = 0.0062$.

(3 marks)

(c) Suppose the joint probability distribution function of X and Y is represented by the following table

X / Y	1	2	3	4
0	0.059	0.1	0.05	0.001
1	0.093	0.12	0.082	0.003
2	0.065	0.102	0.1	0.01
3	0.05	0.75	0.07	0.02

Required (i) Find the marginal probability distribution of X and Y. (2 marks)

(ii) Find $E[Y/X=3]$ and var $[Y/X=3]$ (4 marks)

(d) Suppose that X and Y are two continuous random variables with joint density functions

$$f(x,y)=\begin{cases} kx^3y^3 0\le x\ge 2, 0\le y\le 2 \\ o, otherwise \end{cases}$$

Required

(i) Find the value of k. (2 marks)

(ii) Determine whether variables $x \wedge y$ are independent. (4 marks)

(iii) Find $P[x\le\frac{1}{2}, y>1]$ (2 marks)

(e) Given the covariance matrix of $x \wedge y$

$$\sum ¿ \begin{bmatrix} 3 & \frac{1}{3} \\ \frac{1}{3} & 2 \end{bmatrix}$$ Compute

(i) Variance $[3x+4y-5]$ (2 marks)

(ii) Correlation coefficient of $x \wedge y$ (2 marks)

(f) Let $x \wedge y$ have a bivariate normal distribution with parameter.

$$\mu_x=4 \left| \sigma_x=¿2 \right| e=\frac{3}{5}$$
$$\mu_y=6 \left| \sigma_y=¿15 \right|$$

Compute $p=[12<y<30/x=8]$ (4 marks)

(g) The joint generating function of $f(x,y)$ is given as $\frac{1}{4}e^{t_1}+\frac{3}{4}e^{t_2}¿^6$ $M(t_1,t_2)=¿$

Find (i) Cov (x,y) (2 marks)

(ii) Var (x) (1 mark)

QUESTION TWO (20 MARKS)

(a) Suppose the random variable y has p.d.f.

$f(y)=\begin{cases} 3y^2 0<y<1 \\ 0, otherwise \end{cases}$ find the p.d.f. of $U=27y+3.$ (5 marks)

(b) Let x_1 and x_2 have joint p.d.f.

$$f(x_1,x_2)=\begin{cases} 8x_1x_2, 0<x_1<x_2<1 \\ 0, otherwise \end{cases}$$

Find the p.d.f. of $y_1=x_1/x_2$ (7 marks)

(c) Suppose Ƶ $n(0,1)$ and $y=2$ Find the distribution of y using the m.g.f. technique.

(8 marks)

QUESTION THREE (20 MARKS)

(a) A random variable x has a poisson distribution with parameter λ. Find the characteristic function $\phi(t)$ and use it to find the mean and variance of X. Assume

$$f(x)=\begin{cases}\frac{\lambda^x e^{-\lambda}}{x!}, x=0,1,2,---¿0, otherwise\end{cases}$$

(10 marks)

(b) Let x_1, x_2 be a random sample from a distribution with density function

$$f(x)=\begin{cases}e^{-x} for\, 0 \leq x < \infty \\ 0, otherwise\end{cases}$$

What is the density function of $Y=min\{x_1, x_2\}$ where Y is non zero? (5 marks)

(c) Let $y_1 < y_2 < \ldots < y_6$ be the order statistics from a random sample of size 6 from a distribution with density function

$$f(x)=\begin{cases}2x, 0<x<1 \\ 0, otherwise\end{cases}$$ find the expected value of y_6 (5 marks)

QUESTION FOUR (20 MARKS)

(a) The joint p.d.f of a two-dimensional random variable (x, y) is given by

$$f(x,y)=\begin{cases}2, 0<x<1, 0<y<x \\ 0, elsewhere\end{cases}$$

(i) Show that $f(x,y)$ is a p.d.f.

(ii) Find the marginal density function of x and y

(iii) Find the conditional density functions of y given $X=x$ and of X given $Y=y$

(iv) Check the independence of x and y. (10 marks)

(b) Suppose that x_1 and x_2 are independent random variables and that the p.d.f. of each of these variables is as follows.

$$f(x)=\begin{cases} e^{-x} x \geq 0 \\ 0\, elsewher \end{cases}$$

Find the p.d.f. $y_1 = x_1 + x_2$ (10 marks)

QUESTION FIVE (20 MARKS)

(a) Let x and y be random variables with joint density function

$$f(xy)=\begin{cases} 4xy\, 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0\, otherwise \end{cases}$$

Find (i) $E(x)(ii)E(y)(iii)E(x+y)(iv)E(xy)(v)Cov(xy)$ (7 marks)

(b) Given the joint p.d.f. as

Y X	1	2	3
1	$\frac{1}{12}$	$\frac{1}{6}$	0
2	0	$\frac{1}{9}$	$\frac{1}{5}$
3	$\frac{1}{18}$	$\frac{1}{4}$	$\frac{2}{15}$

Find (i) The marginal distributions of x and y (4 marks)

(ii) The conditional distribution of $Y/x=3$ (3 marks)

(iii) Find $p(x \leq 2, y=3)$ (1 mark)

(iv) Find var ($Y/x=3¿$ (5 marks)
