CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION (ARTS AND SCIENCE), BACHELOR OF SCIENCE (GENERAL) AND BACHELOR OF ARTS (GENERAL)

MATH 315: COMPLEX ANALSIS

STREAMS: BED (SCI & ARTS), BSC (GEN), BA (ECON & MATHS)TIME: 2 HOURS

DAY/DATE: WEDNESDAY 11/4/2018 11.30A.M. – 1.30 P.M.

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

(a) Given that ω_n is an nth root of unity, show that $1+\omega_n+\omega_n^2+\omega_n^{n-1}=0$ [3 marks]

(b) Find the image of $|z-3i|=4$ under the mapping $w=\frac{1}{z}$ [3 marks]

(c) Solve the equation $\cos z=-2$ [4 marks]

(d) Show that the function $f(z)=\frac{\sin z}{z}$ has a removable singularity at $z=0$ [3 marks]

(e) Evaluate the following complex integrals

(i) $\int \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)(z+4)}\, dz$where C is the circle $|z|=3$ [3 marks]

(ii) $\int \frac{e^{2z}}{(2z+1)^4} dz$where C is the circle |z-1|=3 [3 marks]

(f) Find the locus of the points z in the complex plane such that $|z-2i|=|z+2|$ [3 marks]

(g) Find the analytic function $w=f(z)$ from its known real part $u(x,y)=2e^x\cos y$ [4 marks]

(h) Find the Maclaurin's series for the function $f(z)=\sin z$ [3 marks]

QUESTION TWO (20 MARKS)

(a) (i) Given that $z = re^{i\theta}$, show that $Re[log(z-1)] = \frac{1}{2}\log(1 - 2r\cos\theta + r^2)$ for $z \neq 1$. [4 marks]

(ii) Show that the equation above satisfies the Laplace equation for $z \neq 1$.[4 marks]

(b) Determine the region of convergence of the series $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 4^n}$ [5 marks]

(c) Prove that the polar form of the Cauchy-Riemann equations can be written as $\frac{\partial u}{\partial r} = \frac{1\partial v}{r\partial\theta}$ $\frac{\partial v}{\partial r} = \frac{1\partial u}{r\partial\theta}$ [7 marks]

QUESTION THREE (20 MARKS)

(a) (i) State and prove the Cauchy's theorem of integration.

(ii) Hence show that $\oint_c \frac{e^{2z}}{z^2-3} dz = 0 \; where \; C \coloneqq |z+2i| = 1$ [7 marks]

(b) Evaluate the integral $\frac{1}{2\pi i}\int \frac{e^{zt}}{z^2(z^2+2z+2)}\, dz$ around the circle c with the equation $|z| = 3$

[5 marks]

(c) State without proof the residue theorem and use it to evaluate the integral

$\int_c \frac{z^2+5}{(z+2)^3(z^2-5)}\, dz \; C; \; |z-2i| = 6$ [8 marks]

QUESTION FOUR (20 MARKS)

(a) Let z_1, z_2 and z_3 are complex numbers, show that;

(i) $\left||z_1|-|z_2|\right| \leq |z_1 + z_2| \leq |z_1| + |z_2|$ [6 marks]

(ii) $|z_2| \neq |z_3|$ then $\frac{|z_1|}{|z_2+z_3|} \leq \frac{|z_1|}{||z_2|-|z_3||}$ [2 marks]

(b) Show that $\sin^{-1}z = -i\log\left[iz + (1-z^2)^{\frac{1}{2}}\right.$ and $cos^{-1}\, z = -i\; log\left[z + i(1-z^2)^{\frac{1}{2}}\right.$ hence find $tan^{-1}z$ [7 marks]

(c) Verify that the function $f(z) = x^3 \sin y - 3xy^2 + i(3x^2y - y^3 \cos 2x$ is not analytic.

[5 marks]

QUESTION FIVE (20 MARKS)

(a) Differentiate poles, non-isolated singularities and essential singularity using an appropriate example in each case. [6 marks]

(b) Expand the function $f(z) = \cos z \; about \; z = \frac{\pi}{4}$ [4 marks]

(c) Find the Laurent series for the function $f(z) = \frac{e^{2z}}{(z-1)^3}$ about $z = 1$ and identify the nature of singular point at $z = 1$ [4 marks]

(d) Expand the function $f(z) = \frac{1}{(z+1)(z+3)}$ using a Laurent series about

(i) $1 < |z| < 3$

(ii) 0$< |z + 1| < 2$ [6 marks]
