CHUKA **UNIVERSITY**

UNIVERSITY SUPPLEMENTARY/SPECIAL EXAMINATIONS.

FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE (GENERAL)

MATH 401: MEASURE THEORY

STREAMS: BSC (GENERAL) **TIME: 2 HOURS**

DAY/DATE: TUESDAY 24/07/2018 **11.30 A.M - 1.30 P.M**

INSTRUCTIONS:

- **Answer Question ALL the Questions**

QUESTION ONE: [30 MARKS]

a) Define the following:

(i)	A Lebesque- measurable subset E of R	[2 Marks]
(ii)	An algebra of a non-void set X	[2 Marks]
(iii)	σ- algebra	[1 Mark]
(iv)	A measurable space	[1 Mark]
(v)	A measure space	[1 Mark]

b) Let $\mu^{¿}(A)=0$, show that $\mu^{¿}(A\cup B))=\mu^{¿}(B)$ [5 Marks]

c) Define a measurable function and show that a constant function on a measurable set is measurable. [5 Marks]

d) Define the characteristic function on a measurable subset E of R and show that it is measurable [5 Marks]

e) Define the probability measure. [4 Marks]

f) State the monotone convergence theorem. [4 Marks]

QUESTION TWO: [20 MARKS]

a) Prove that if E is non-Lebesque measurable subset of R, then there exists a subset $A$of E such that $0<M^{¿}[A]<\infty$ [6 Marks]

b) Show that if $\mu*(A)=0$, then A is measurable hence or otherwise show that the set of rational numbers is measurable. [6 Marks]

c) Let A be a Lebesgue measurable subset of R, and B be any other subset of R. show that $\mu*(A\cup B)+\mu*(A\cap B)=\mu*(A)+\mu*(B)$. [8 Marks]

QUESTION THREE: [20 MARKS]

a) Prove the following properties of $\mu*¿¿$

i. $\mu^{¿}(\varphi)=0$ [2 Marks]

ii. $\mu^{¿}(\{x\})=0$ [3 Marks]

iii. If $A\subseteq B$ then $\mu*(A)\leq\mu*(B)$ [3 Marks]

iv. $\mu*¿¿$ is countablysubadditive [4 Marks]

b) Let (X,x) be a measurable space and $f: X\rightarrow R^{¿}$ be a given function. Show that the following statements are equivalent

i. $\{x\in X: f(x)>a\}(¿f^{-1}¿)\in x$ for all $a\in R^{¿}$

ii. $\{x\in X: f(x)\geq a\}(¿f^{-1}[a,\infty])\in x$ for all $a\in R^{¿}$

iii. $\{x\in X: f(x)<a\}(¿f^{-1}¿)\in x$ for all $a\in R^{¿}$

iv. $\{x\in X: f(x)\leq a\}(¿f^{-1}[-\infty,a])\in x$ for all $a\in R^{¿}$ [8 Marks]

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