MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2018/2019 ACADEMIC YEAR

FIRST YEAR **SECOND** SEMESTER EXAMINATION FOR, BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING

SMA2200 – CALCULUS III

DURATION: 2 HOURS

DATE: 23RD APRIL 2019

TIME: 9.00 – 11.00 A.M

Instructions to candidates:

1. Answer question One and Any Other Two questions
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a. Explain what is meant by a limit of a sequence. [2 Marks]

b. Find a formula for the nth partial sum S_n given the series.

$\sum_{n=1}^{\infty} \frac{2n}{3n-2} - \frac{2n+4}{3n+4}$ is the series convergent or divergent? [5 Marks]

c. Use series expansion to evaluate $\lim_{x \to 0} \frac{e^x + x - 1}{\sin x}$. [4 Marks]

d. Determine the interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n4^n}$ [5 Marks]

e. Evaluate $\int x^5 (\ln x)^2 dx$ using reduction formula . [3 Marks]

f. Evaluate $\int_{\infty}^{\infty} \frac{dx}{x^2 + 4x + 5}$ [5 Marks]

g. Find the Fourier series expansion of $f(x) = 5x + 1$ for $-1 \le x \le 1$ [6 Marks]

QUESTION TWO (20 Marks)

a. **Show that** $n\int \cos^n xdx = \sin x \cos^{n-1} x + (n+1)\int \cos^{n-2} xdx$ **and evaluate** $\int_0^{\frac{\pi}{2}} \cos^4 xdx$ **.**

[9 Marks]

b. Compute $\int_0^4 \int_0^{\sqrt{4-y}} \left(4 - x^2 - y\right) dxdy$ [5 Marks]

c. **Show that if** $f(x, y, z) = \ln\left(x^2 + y^2 + z^2\right)^{\frac{1}{2}}$**, then** $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$ [6 Marks]

QUESTION THREE (20 Marks)

a. Find the Maclaurin's series expansion for the function $y(x) = \sin x$ up tothe term in x^3 [4 Marks]

b. Evaluate $\lim_{x \to 0} \left[\frac{1 - \sqrt{1 + 2x}}{1 - \sqrt{1 - 3x}} \right]$ using binomial expansion. [6 Marks]

c. Show that the series $\sum_{k=2}^{\infty} 3\left(-\frac{1}{5}\right)^{k}$ converges and find its sum. [5 Marks]

d. Find the volume of the resulting solid rotated about the x – axis for the region R bounded by the curves $y = x$ and $y = x^2$. [5 Marks]

QUESTION FOUR (20 Marks)

a. Find the area of the surface generated by rotating one arc of the cycloid $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$ about the x – axis. [5 Marks]

b. Given $v = \left(\sqrt{x^2 + y^2 + z^2}\right)^{-1}$, show that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$ [7 Marks]

c. By interchanging the order of integration evaluate the integral $\int_0^1 \int_{-1}^3 (3y + x)\,dxdy$ [4 Marks]

d. Evaluate $\int_0^{\infty} \frac{1}{4 + x^2} dx$ and determine whether the integral is convergent. [4 Marks]