DEPARTMENT OF PHYSICS

SPH 800 CLASSICAL MECHANICS TAKE AWAY CAT: IBP AUGUST 2019

Answer ALL questions

Q1 a) Derive Lagrange's equations from Hamilton variational principle [3 marks]

b) Use Hamilton's variational principle to obtain equation of motion of a simple pendulum [4 marks]

c) Two identical masses are connected by a spring of spring constant k. An identical spring attaches one mass to a fixed wall. The system is free to move on a frictionless horizontal surface. Use Lagrange's equations to to obtain the equation of motion of the system. [5 marks]

Q 2. a) The position vector of a molecule is given in cylindrical coordinates as

$$\vec{r} = \rho\hat{\rho} + z\,\hat{k}$$

Obtain the expression of the kinetic energy of the molecule. [3 marks]

b) If the electron moves in a spherically symmetric force field in which the potential energy is given by

$$V = \frac{\lambda \cos\theta}{r^2}$$

where λ is a constant. Determine:

i) The Hamiltonian function of the electron in spherical coordinates. [5 marks]

ii) The equations of motion of the system by Hamilton's method [5 marks]

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Q3. i) A function is given as $F = F(x, \acute{x}, y)$ where $\acute{x} = \frac{dx}{dy}$. Write down the Euler – Lagrange equation of the system. What is the physical significance of this equation? [2 marks]

ii) Find the curve $x = x(y)$ which minimizes the function

$$J = \int(\acute{x}^2 + 3)dy$$

given that $x(0) = 3$ and $x(2) = 5$ [3 marks]

Q4. a) i) What is meant by a canonical transformation ? [1 mark]

ii) Given that the first(F_1) and the fourth (F_4) generating functions are related by

$$F_4 = F_1 + \sum P_k Q_k - \sum p_k q_k$$

Derive the transformation equations for the fourth type generating functions [4 marks]

b) i) Prove that the transformation $q = \sqrt{\frac{2P}{\sqrt{km}}} \sin Q$ and $p = \sqrt{2P\sqrt{km}} \cos Q$ is canonical. [3 marks]

ii) Using the transformation given in b(i) express the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}kq^2$ of a simple harmonic oscillator in terms of Q and P and show that Q is cyclic. [3 marks]

iii) Obtain the solution of the given harmonic oscillator by using the above results. [3 marks]

b) Consider any two continuous functions $F(q_k, p_k, t)$ and $G(q_k, p_k, t)$ of the generalized position coordinates, momenta, and time,

i) Define the Poisson bracket for the function F and G. [1 mark]

ii) If H is the Hamiltonian, prove that :

$$\frac{dF}{dt} = [H, F] + \frac{\partial F}{\partial t}$$ [3 marks]

Q5. A particle of mass m is sliding down an incline plane whose lower end is making an angle θ with the horizontal. Taking the displacement of the particle from the top of the incline as x, solve for the motion of the particle using Hamilton – Jacobi method. [6 marks]

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