W1-2-60-1-6

Jomo Kenyatta University of Agriculture and Technology

University Examinations 2017/2018

SECOND/THIRD YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREES OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE, BACHELOR OF SCIENCE IN BIOSTATISTICS, BACHELOR OF SCIENCE IN BUSINESS COMPUTING, BACHELOR OF SCIENCE IN COMPUTER SCIENCE, BACHELOR OF SCIENCE IN FINANCIAL ENGINEERING, BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY, BACHELOR OF SCIENCE IN MATHEMATICS, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE IN OPERATIONS RESEARCH, AND BACHELOR OF SCIENCE IN STATISTICS

STA 2200 PROBABILITY AND STATISTICS II

DATE: JANUARY 2018 DURATION 2 HOURS

INSTRUCTIONS:

(1) Answer Question ONE and any other two questions.
(2) NO SMP tables are allowed, but you may use a non-pre-programmable calculator, and a set of statistical tables has been provided.
(3) NO writing is allowed on any paper apart from the answer booklet. In particular marks will be deducted if any writing is done on the question paper or on the statistical tables.
(4) The question paper and the statistical tables should be placed inside the answer booklet, when this is handed in. Failure to comply will result in deduction of marks
(5) Mobile phones, i-pads, tablets and any other illegal materials are prohibited in the examination room.

QUESTION ONE (COMPULSORY) (30 MARKS)

(a) Six percent of a large batch of light bulbs is faulty. Dr Patrick Kihato buys a set of 8 bulbs at random. Determine correct to three decimal places

(i) The probability that there is no faulty bulb.

(ii) The probability that there are exactly two faulty bulbs.

(iii) The probability that there are at least two faulty bulbs. (2,2,3)

(b) The random variable X has mean 20 and variance 12. Determine the mean and the variance of the random variable Y = 120 - 4 X. (4)

(c) The continuous random variable X has probability density function f(x) given by:

$$f(x) = \begin{cases} \frac{k}{x} & for\ 1 < x < 2 \\ 0 & elsewhere. \end{cases}$$

(i) Show that the constant $k = \frac{1}{\ln 2}$.

(ii) Show that the median M of the distribution is given by $M = \sqrt{2}$. (2,3)

(d) Cartons of orange juice are marked '500 grams' but the masses are normally distributed with mean 515 grams. Determine the standard deviation of the masses of the cartons if only 2.3% of the cartons have a mass less than 500 grams. (4)

(e) Masses of form 4 students in Kwacha county in the country of Burania have a known variance of 36 Kg^2. A sample of 120 students is taken at random, and their mean mass was found to be 63.5 Kg. Determine correct to two decimal places the 95 % confidence interval for the true mean value μ of the masses of the students. (4)

(f) A continuous random variable X has a moment generating function $M_x(t) = \frac{1}{(1-3t)^2}$.

Use the moment generating function to determine E(X) and Var (X). (6)

QUESTION TWO (20 MARKS)

(a) A discrete random variable X has the probability distribution as given in the table below.

x	0	1	2	3	4
P (X = x)	0.1	a	0.5	b	0.1

Given that E (X) = 2.1, determine the unknown probabilities a and b, and also the variance of the distribution. (5,2)

(b) The number of patients waiting at any time in a doctor's surgery hour has a Poisson distribution with mean number of patients being 2.5. Determine correct to three decimal places the probability that :

(i) There is no patient waiting.

(ii) There is exactly one patient waiting.

(iii) There are more than two patients waiting. (2,2,3)

(e) A jeweller has 12 gold rings and 8 silver rings in his stock. A travelling salesman has arranged to carry 5 rings at random to display to his customers. Determine correct to three decimal places:

(i) The probability that he will carry 3 gold and 2 silver rings.

(ii) The probability that he will carry at least 3 gold rings. (2,4)

QUESTION THREE (20 MARKS)

The time in years to failure of an expensive machine can be modelled by a random variable T with PDF f(t) given by: $f(t) = \begin{cases} \frac{1}{10}e^{-\frac{t}{10}} & for\ t > 0 \\ & elsewhere. \end{cases}$

(a) Determine the CDF F(t). (2)

(b) Determine correct to three decimal places the probability that the time to failure lies between four and six years. (3)

(c) Determine correct to three decimal places the probability that the machine is still working after two years. (2)

(d) Determine correct to three decimal places the probability that the machine is still working after seven years, GIVEN that it is still working after five years. What do you observe about your answers for parts (c) and (d)? (4)

(e) Show that the lower quartile of the distribution is $10\ell n\left(\frac{4}{3}\right)$. (3)

(f) Determine the mean time to failure, and also the probability correct to three decimal places that the machine is still working after the mean time to failure. Determine the mode of the distribution. (4,2)

QUESTION FOUR (20 MARKS)

(a) Every secretary in a large company has been given the same assignment. The time to completion is normally distributed with mean time 90 minutes and standard deviation 12 minutes.

(i) Determine the probability that a secretary will take longer than 110 minutes to complete the assignment. (3)

(ii) Determine the probability that a secretary will take between 85 and 96 minutes to complete the assignment. (5)

(iii) For the 20 % of the secretaries who finish the assignment in the shortest time, the directors have provided a free lunch. Determine to the nearest minute the maximum time to completion in order for the secretary to qualify for the free lunch. (4)

(b) A mobile phone company claims that the batteries on their phones have an average 'life' of 31.5 hours before a charge is necessary. The 'lives' are normally distributed. The batteries on six phones were tested and their 'lives' in hours were 29, 28, 32, 30, 26 and 35. Test at the 5 % level of significance whether these batteries have a mean life that is shorter than the mean 'life' claimed by the company. (8)