



**JOMO KENYATTA UNIVERSITY
OF
AGRICULTURE AND TECHNOLOGY**

UNIVERSITY EXAMINATIONS 2016/2017

**FIRST YEAR EXAMINATIONS FOR BACHELOR OF SCIENCE IN: ACTUARIAL
SCIENCE/ANALYTICAL CHEMISTRY/BIOSTATISTICS/ CONTROL &
INSTRUMENTATION/ COMPUTER SCIENCE/ INFORMATION
TECHNOLOGY/FINANCIAL ENGINEERING/ MATHEMATICS & COMPUTER
SCIENCE/ SCIENCE/ STATISTICS**

SMA 2100: DISCRETE MATHEMATICS

DATE: DECEMBER 2016

TIME: 2 HOURS

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY)

- a) List the members of the set $A = \{x \in \mathbb{N} \mid 4 + x = 3\}$ [1 mark]
- b) If $S = \{0, 1, 2, 3\}$, find $P(S)$, the power set of S . [4 marks]
- c) Prove that if n is an integer and $3n + 2$ is even, then n is even using
 i. A proof by contraposition [3 marks]
 ii. A proof by contradiction [3 marks]
- d) Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $f(x) = 2x + 1$, determine if f is invertible [4 marks]
- e) Write the converse, inverse and contrapositive of the following statement
 "If $1 + 1 = 2$, then pigs can fly" [6 marks]
- f) Let $P(x)$ be the statement " $x = x^2$." If the domain consists of the integers, what are these truth values?
 i) $P(1)$ ✓
 ii) $\exists x P(x)$ ✓
 iii) $\forall x P(x)$ ✗ [3 marks]
- g) Using a Venn diagram to show that $\overline{A \cup B} = \bar{A} \cap \bar{B}$, if A and B are sets [4 marks]
- h) Find the domain D of the real-valued function, $g(x) = \sqrt{25 - x^2}$ [2 marks]

2016/12/16
(L017L017)

$x=3-4$

RUSIN

FC Karamba

2+1

$$\begin{aligned} \sqrt{25-x^2} &= 0 \\ 25-x^2 &= 0 \\ 25-x^2 &= 0 \\ 25-x^2 &= 0 \\ 25-x^2 &= 0 \end{aligned}$$

QUESTION TWO

- a) In a survey of 120 people, it was found that: 65 read Newsweek magazine, 20 read both Newsweek and Time, 45 read Time, 25 read both Newsweek and Fortune, 42 read Fortune, 15 read both Time and Fortune, 8 read all three magazines.
- i) Draw a Venn diagram with N , T , and F denoting the set of people who read Newsweek, Time, and Fortune, respectively and fill in the correct number of people in each of the eight regions of the Venn diagram [4 marks]
 - ii) Find the number of people who read at least one of the three magazines [2 marks]
 - iii) Find the number of people who read exactly one magazine. [2 marks]
- b) Determine whether $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are logically equivalent [12 marks]

QUESTION THREE

- a) Prove the following by mathematical induction
- i) $7^n - 2^n$ is divisible by 5 for all $n \in \mathbb{N}$ [4 marks]
 - ii) $2 + 4 + 6 + \dots + 2n = n(n+1)$ [4 marks]
- b) Using a Venn diagram with sets A , B and C , shade the following sets:
- i) $A - (B \cup C)$ [2 marks]
 - ii) $\bar{A} \cap (C - B)$ [2 marks]
- c) Let $S = \{a, b, c, d, e\}$, $T = \{1, 2, 3, 4\}$. State with reasons whether each of the following statements is either true or false.
- i) It is possible to define an injection $f: S \rightarrow T$ [2 marks]
 - ii) It is possible to define a function $f: T \rightarrow S$ [2 marks]
- d) Find the Cartesian product $A \times B \times C$ where $A = \{0, 1\}$, $B = \{1, 2\}$ and $C = \{0, 1, 2\}$ [4 marks]

0 1 0 0 1 0
0 1 1 0 2 1
0 1 2 0 2 2

QUESTION FOUR

- a) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Find the formula for the composition functions $g \circ f$, $f \circ g$ and $f \circ f \circ g$ [7 marks]
- b) Prove that $\sqrt{5}$ is irrational by contradiction [6 marks]
- c) Using set identities, show that $(A \cap B) \cup (A \cap \bar{B}) = A$ [4 marks]
- d) Let p , q , and r be the propositions
 p : You have the flu.
 q : You miss the final examination.
 r : You pass the course
 Express the proposition $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$ as an English sentence [3 marks]

$7(5t + 2k) - 2^k \sqrt{2}$
 $35t + 14k - 2^k \times 2$

$k(k+1) + (2k+1)$
 $k^2 + k + 2k + 1$
 $k^2 + 3k + 1$
 $7^{k+1} - 2^{k+1}$
 $7^k + 7 - 2^k - 2$

TTT
TF
TF
FT
F
F
F
F
TTT
TF
TF
FT
F
F
F
F
1
2
3
4