JOMO KENYATTA UNIVERSITY
OF
AGRICULTURE AND TECHNOLOGY
UNIVERSITY EXAMINATION    2016/2017 ACADEMIC YEAR

*FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR
OF SCIENCE IN OPERATIONS RESEARCH, BIOSTATISTICS, STATISTICS,
FINANCIAL EGNEERING, AND CTUARIAL SCIENCE*

### STA 2101 ALGEBRA FOR STATISTICS AND FINANCE

DATE: AUGUST, 2016                                                    TIME 2 HOURS

**Instructions**  i) Answer question one and nay other two questions.
ii) Strictly submit this question paper together with the answer booklet

## QUESTION ONE (30 MARKS)

a) Solve the system of equation $\begin{matrix} 2x - 3y = 7 \\ 4x + 3y = 5 \end{matrix}$  using matrix                (3 marks)

b) Given that $f(x) = 2 + 3x - x^2$ and $g(x) = 2x - ?$ evaluate;

   (i)    $f \circ g(x)$                                                              (1 Marks)

   (ii)   $g \circ f(3)$                                                             (1 Marks)

c) $y$ varies jointly with $x^3$ and $z$, and varies inversely with $y^2$. What is the effect on $y$ when $x$ is doubled and $r$ is halved?                (3 Marks)

d) Factorize the expression $f(x) = x^3 - 1$                                       (2 Marks)

e) Estimates of the amounts (in $ billions) of
U.S. Online advertising spending from 2007
through 2011 can be modelled by an
exponential growth model $S = 10.33e^{0.1022t}$,
$7 \le t \le 11$, where S is the amount of spending
(in billions) and $t = 7$ represents 2007. Find
when the amount of U.S. online advertising
spending will reach $40 billion.    (4 marks)

f) Write the cubic function whose graph is
shown alongside                (3 marks)

g) What are the asymptote of the Graph $y = \frac{2}{x+3} - 1$? Hence sketch the graph        (3 marks)
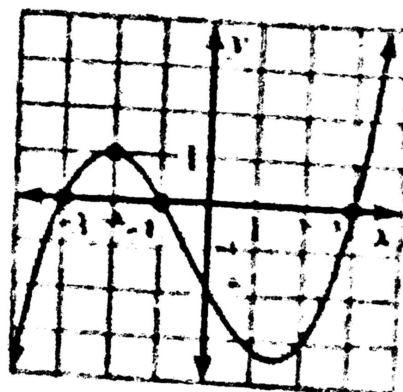
h) Describe the graph of the equation $\frac{x^2}{4}+\frac{y^2}{9}=1$ (4 marks)

i) Graph the hyperbola $25x^2-9y^2+225=0$ (3 marks)

j) Find the equation of the ellipse centered at $(-4,3)$ with minor axis of length 6 and with foci at $(-4,3\pm4)$ (3 marks)

## QUESTION TWO (30 MARKS

a) The average number of phone calls per day between two cities has been found to be jointly proportional to the populations of the cities, and inversely proportional to the square of the distance between the two cities. The population of Charlotte is about 1,500,000 and the population of Nashville is about 1,200,000, and the distance between the two cities is about 400 miles. The average number of calls between the cities is about 200,000.

i) Find the proportionality constant k and write the equation of variation. (2 Marks)

ii) The average number of daily phone calls between Charlotte and Indianapolis (which has a population of about 1,700,000) is about 134,000. Find the distance between the two cities. (4 Marks)

b) Determine all the roots of the polynomial function $p(x)=x^4+x^3-11x^2-5x+30$ (7 Marks)

c) Solve the system of equation using crammers rule (7 Marks)

$$3(x+2y-z=2$$

$$\begin{bmatrix} 3x+6y+z=1 \\ 3x+3y+2z=3 \end{bmatrix}$$

## QUESTION THREE (20 MARKS)

i) Write an exponential function of the form $y=ub^x$ whose graph passes through the points (1, 4), (3, 16). (3 marks)

ii) The population of a city is $P=250,342e^{0.012t}$ where $t=0$ represents the population in the year 2000.

i) Find the population of the city in the year 2010 and in the year 2015. (2 marks)

ii) Find the year when the population will be 320,000. (3 marks)

iii) A rainbow trout can grow up to 40 inches in length. The weight y (in pounds) of a rainbow trout is related to its length x (in inches) according to the model $y=0.0005x^3$. Graph the model. Use your graph to estimate the length of a 10 pound rainbow trout. (Hint; $0\le x\le40$ use steps of 5 inches in x) (6 marks)

d) The table shows the population y (in millions) and the population rank x for nine cities in Argentina in 1991.

i) Draw a scatter plot of $\ln y$ versus $\ln x$. Is a power model a good fit for the original data?

ii) Find a power model for the original data. Estimate the population of the city Vicente López, which has a population rank of 20 (6 marks)

| City | Rank x | Population (millions), y |
|---|---|---|
| Cordoba | 2 | 1.21 |
| La Matanza | 3 | 1.12 |
| Mendoza | 4 | 1.11 |
| La Plata | 5 | 0.77 |
| Moron | 6 | 0.64 |
| San Miguel de Tucuman | 7 | 0.64 |
| Tucuman | 8 | 0.62 |
| Lomas de Zamoras | 9 | 0.57 |
| Mar de Plata | 10 | 0.51 |

## QUESTION FOUR (20 MARKS)

a) Find the centre and the radius of the circle described by the equation $x^2 + y^2 - 6x - 12y - 55 = 0$

(3 marks)

b) The lines $y = \dfrac{4}{3}x - \dfrac{5}{3}$ and $y = -\dfrac{4}{3}x - \dfrac{13}{3}$ pass through the centre of a circle and the point $(-5, 0)$ is on that circle. Find the equation of this circle. (4 marks)

c) Sketch and describe the graph given by the equation $4x^2 + 9y^2 - 16x - 54y + 61 = 0$

(7 marks)

d) Sketch and describe the graph of the hyperbola $16(x+3)^2 - 4(y-2)^2 = 64$ (6 marks)