WI-2-60-1-6

JOMO KENYATTA UNIVERSITY

OF

AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2017/2018

SECOND YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE, BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF SCIENCE IN FINANCIAL ENGINEERING, BACHELOR OF SCIENCE IN OPERATIONS RESEARCH, BACHELOR OF SCIENCE IN BIOSTATISTICS, BACHELOR OF SCIENCE GENERAL (PHYSICAL SCIENCE) & BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

STA 2201: PROBABILITY AND STATISTICS III

DATE: AUGUST 2018 **TIME: 2 HOURS**

INSTRUCTIONS TO CANDIDATES:

1. *Answer questions ONE and any two questions*
2. *Be neat and show all your workings*
3. *All questions except question one carry equal marks*

This paper consists of 4 printed pages

STACS Examination board 2017/2018

QUESTION ONE (30 MARKS)

a) State two conditions for a function f(x, y) to be a joint probability density function of the continuous random variables X and Y. (2 Marks)

b) Let Y_i, i = 1, 2, 3, 4 be independently distributed normal random variables with means 1 and variances i. state the distributions of the following random variables

i) $V = Y_1 - Y_3 - Y_4$ (1 Mark)

ii) $$U = (Y_1 - 1)^2 + \left(\frac{Y_3 - 1}{\sqrt{3}}\right)^2 + \left(\frac{Y_4 - 1}{2}\right)^2$$ (1 Mark)

iii) $$X = \frac{(Y_1 - 1)}{\sqrt{\left[(Y_1 - 1)^2 + \left(\frac{Y_3 - 1}{\sqrt{3}}\right)^2 + \left(\frac{Y_4 - 1}{2}\right)^2\right]\frac{1}{3}}}$$ (1 Mark)

iv) $$W = \frac{\left(\frac{Y_2 - 1}{\sqrt{2}}\right)^2}{\sqrt{\left[(Y_1 - 1)^2 + \left(\frac{Y_3 - 1}{\sqrt{3}}\right)^2\right]\frac{1}{2}}}$$ (1 Mark)

c) Given that $f(x,y) = \begin{cases} k(2x+y) & ,0<x<2, 0<y<2 \\ 0 & otherwise \end{cases}$

is a joint probability distribution of continuous random variables X and Y determine the value of the constant k and hence compute P(X>Y). (5 Marks)

d) If X and Y are random variables, show that
$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2ab Cov(X,Y)$, where a and b are constants. (4 Marks)

e) Suppose that the joint probability distribution of the discrete random variables is given by

$$f(x,y) = \begin{cases} \frac{1}{9}, & x = 1,2,3,\ y = 1,2,3 \\ 0 & otherwise \end{cases}$$

i) Obtain the joint moment generating function of X and Y. (2 Marks)

$\frac{1}{9} e^{tx} \cdot e^{ty}$

$e^{t_1 x + t_2 y}$

$\frac{1}{9} e^{t} + \frac{1}{9} e^{t}$

ii) Determine the marginal moment generating functions of X and Y. (2 Marks)

iii) Determine whether X and Y are independent. (3 Marks)

f) Suppose the parameter of bivariate normal density of random variables of X and Y are

$\mu_x = 4,\ \mu_y = 6, \sigma_x^2 = 4, \sigma_y^2 = 8, \rho_{xy} = \frac{1}{2\sqrt{2}}$

Determine

i) the conditional probability distribution of Y given X= x, (3 Marks)

ii) the regression of Y ~~and~~ on X, (1 Mark)

iii) the conditional variance of Y given variance of Y given X= x. (1 Mark)

g) Each of two bags contains three identical balls marked 1 to 3. Two balls are drawn at random, one from each bag, simultaneously. Let the random variable X represent the maximum of the numbers on the balls drawn and Y the minimum of numbers on the balls drawn. Determine the joint probability distribution of X and Y. (3 Marks)

QUESTION TWO (20 MARKS)

a) Let X and Y be independent random variables such that X has the gamma density $\Gamma(\alpha_1, \beta)$ and Y has the gamma density $\Gamma(\alpha_2, \beta)$ Let V = X+ Y and $U = \frac{Y}{X+Y}$ be two new random variables. Obtain the joint p.d.f's of U and V. (15 Marks)

b) Let X be a standard normal random variable i.e. X~ N(0, 1) and U be a chi – square random variable with n degrees of freedom i. e U ~ $\chi^2_{(n)}$. Assuming that X and U are stochastically independent, obtain the mean and variance $T = \frac{X}{\sqrt{\frac{U}{n}}}$. (5 Marks)

QUESTION THREE (20 MARKS)

a) Let $X_1, X_2, \ldots, X_n$ be a sample of size n ≥ 2 from a distribution that is normally distributed with mean μ and variance σ^2. Determine the distributions of $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and

$\frac{nS^2}{\sigma^2}$, where $S^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$ (11 Marks)

b) The joint probability distribution of two discrete random variables X and Y is given by

X \ Y	0	1	2	$f_1(x)$
0	0.1	0.2	0	0.3
2	0.4	0.1	0	0.5
3	0.1	0	0.1	0.2
$f_2(y)$	0.6	0.3	0.1	

i) Obtain the marginal p. d. f's of X and Y (2 Marks)

ii) Find the p.d. f of Y given X=2. (4 Marks)

iii) Are X and Y independent? Why? NO (3 Marks)

QUESTION FOUR (20 MARKS).

a) Suppose that the random variables X and Y have a continuous joint p. d. f given

$$f(x,y)=\begin{cases}\frac{1}{2}xy & ,0<x<y<1\\ 0 & otherwise\end{cases}$$

Compute E (X), E(Y), Var (Y), Cov (X,Y) and ρ_{xy} . Hence comment whether X and Y are positively or negatively correlated. (13 Marks)

b) The joint p. d. f of X and Y is given by

$$f(x,y)=\begin{cases}x+y & ,0<x<1,0<y<1\\ 0 & otherwise\end{cases}$$

Determine the joint distribution function (c.d. f) of X and Y. hence or otherwise compute P (0< X < ½ , ½ < Y < ¾) (7 Marks)

$$\int_0^x\int_0^y (x+y)\,dy\,dx$$

$$\left[xy+\frac{y^2}{2}\right]_0^y$$

$$\frac{x^2y}{2}+\frac{xy^2}{2}$$

$E(X)=\iint x\, f(x,y)\,dx\,dy$