W1-2-60-1-6

JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

University Examinations 2017/2018

YEAR II EXAMINATION FOR THE DEGREE OF BACHELOR OF OPERATION RESEARCH, FINANCIAL ENGINEERING AND ACTUARIAL SCIENCE

SMA 2231: DIFFERENTIAL EQUATIONS

DATE: AUGUST, 2018 **TIME: 2 HOURS**

INSTRUCTIONS: **Answer Question ONE and Any Other TWO Questions.**

QUESTION ONE (30 MARKS)

a) Explain each of the following

i. Linear differential equation of order n (1 mark)

ii. Total differential of $F(x,y)$ (1 mark)

iii. Solution of a differential equation (1 mark)

Hence find the values of the constants a and k if they exists such that $y = ae^{x} + \cos kx$ is a solution of the equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 3e^{x}$ (3 marks)

b) If money is withdrawn at a constant rate of K£500 per year from an account that earns interest at 10% per year compounded continuously, write down a differential equation satisfied by the amount A in the account at any time t (in years). Hence show that $A = 5000 + Ce^{0.1t}$, given that $A(0) = 4000$ determine the amount in the account after five years (5 marks)

c) Solve each of the following

i. $\left(\frac{3-y}{x^2}\right)dx + \left(\frac{y^2-2x}{xy^2}\right)dy = 0,\ y(-1) = 3$ (3 marks)

ii. $\left(x^3 + y^2\sqrt{x^2+y^2}\right)dx - \left(xy\sqrt{x^2+y^2}\right)dy = 0$ (4 marks)

d) Differentiate between ordinary and regular singular point of the equation $a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$, hence using Taylor's expansion find a power series solution of the initial value problem $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 6y = 0,\ y(1) = 1,\ y'(1) = -6$ (5 marks)

e) Solve the equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4x^2$ (3 marks)

f) Given the system

$$\frac{dx}{dt} + \frac{dy}{dt} + x - 3y = 3t$$

$$\frac{dx}{dt} + 2\frac{dy}{dt} - 2x - 3y = 1$$

, express x as a function of t (4 marks)

QUESTION TWO (20 MARKS)

a) Explain each of the following

i. Exact differential equation (1 mark)

ii. Integrating factor (1 mark)

Hence determine an integrating factor of the equation $\frac{dy}{dx} + p(x)y = Q(x)$ (5 marks)

b) A student has K£600 in his saving account initially, he decides to deposit K£5 from his pocket money each week if the account pays 0.65% compound interest per year. Show that the amount $P(t)$ in the students account at any time t satisfies a linear differential equation. Determine (7 marks)

  i. Amount in the account at any time t (2 marks)
  ii. Balance in the account after two years (4 marks)
  iii. Time it will take for the balance to exceed K£1500

QUESTION THREE (20 MARKS)

a) Find the general solution of each of the following

  i. $x\frac{dy}{dx} + y = -2x^6 y^4$ (4 marks)

  ii. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \frac{e^{-x}}{x}$ (6 marks)

b) An influenza epidemic has spread throughout a community of 50000 people at a rate proportional to the product of the number of people who have been infected and the number who have not been infected. If 100 individuals were infected initially and 500 were infected after 10 days later

  i. How many people will be infected after 20 days (7 marks)
  ii. When will half of the community be infected (3 marks)

QUESTION FOUR (20 MARKS)

a) The Kenya's productivity is approximately given by $F(x,y) = 10x^{0.75}y^{0.25}$ with the utilization of x units of labor and y units of capital.

  i. Find $F_x(600,400)$ and $F_y(600,400)$. Interpret your results (5 marks)
  ii. If the country is presently utilizing 600 units of labor and 400 units of capital find the change in productivity if labor units are increased by 50 units and capital units are increased by 20 units (3marks)

b) Find a series solution in powers of x for the equation

$$\left(x^2+1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$$ (12 marks)

- 0.016096666

$2/1.03$

$2^{2/3}$

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