JKUAT DEPARTMENT OF STATISTICS AND ACTUARIAL SCIENCES

STA 2490 DEMOGRAPHIC TECHNIQUES

B SC YEAR 4 ACTUARIAL SCIENCE/BIOSTASTISTICS CAT 2 DECEMBER 2017

1. The town of Prosperity in the country of Burania had its population counted every ten years when a census was taken. The census recorded the population size as given in the table below.

Year	1965	1975	1985	1995	2005
Population P in thousands	20.0	21.1	22.3	23.5	24.7

A demographer at BNBS in 2014 proposed to fit the data to a model $P = ae^{bt}$ where P is the population in thousands and t is the time in DECADES after the first observation in 1965, to predict the population size in 2015.

(a) Transform the model into appropriate logarithmic form, and fit a regression model to the data to determine the values of the constants a and b, each correct to three decimal places.

(b) Use your model to predict the size of the population (in thousands) in 2015 correct to three significant figures.

(c) The 2015 census figures gave the actual population of Prosperity town as 25,800. Determine the percentage error in your prediction in (b) correct to the nearest 0.1%.

2. A Gompertz population model was used to estimate the population P(t), t years after the initial observation of the population, of the town of Kajanga. The population P(t) is given by: $P(t) = 10{,}000 * 6^{(1\,01^t)}$

(i) Write down the initial observed population size.

(ii) Determine the time t, correct to two decimal places, for the population size to become ~~three~~ times the initial observed population size.

(iii) Determine the instantaneous rate of growth of the population at the time when the population size is three times the initial population size, correct to the nearest 100 persons per annum.

3. A town's population is modelled on the logistic model, and the population P, t years after the initial observation, is given by: $P = \dfrac{180{,}000}{4 + 5e^{-0.04t}}$.

(a) Determine the initial and ultimate sizes of the population.

(b) Show that the graph of P against t has a point of inflection when $e^{-0.04t} = \frac{4}{5}$.

$36{,}000 \times \frac{4}{5}$

$-1 \quad 4+5e^{-t}$

$(4+5e^{-0.04t})^{-1}$ $\quad e^{-0.04t}$ $\quad -2=0$ $\quad (180{,}000)(4+5e^{-0.04t})^{-1}$

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(c) Hence determine the exact value of the rate of change of the population, when the curve for the population has a point of inflection, and show that, when this occurs, the population size will be exactly half the size of the ultimate population.

(d) Determine the value of t at the point of inflection correct to two decimal places.