(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 123

COURSE TITLE: LINEAR ALGEBRA

DATE: 03/08/18 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 Marks)

a) Define the following terms:

i) Trace of matrix (1 mk)

ii) Linear combination of a vector (2 mks)

iii) Transpose of a matrix (1 mk)

iv) Vector space (2mks)

b) Let A and B be invertible matrices. Prove that $(AB)^{-1}=B^{-1}A^{-1}$ (3 marks)

c) Let AX=B be system of linear equation. Show that if A^{-1} exists , the solution is unique and is given by $X=A^{-1}B$ (3marks)

d) Prove that the following transformation hT: $R^2 \rightarrow R^2$ is linear. $\mathbf{T}\ (\mathbf{x}, \mathbf{y}) = (\mathbf{2x}, \mathbf{x} + \mathbf{y})$ (4 mks)

e) Find AB given that

$$A = \begin{bmatrix} Cos\,\theta & -\sin\theta \\ Sin\,\theta & Cos\,\theta \end{bmatrix},\ B = A^T$$

(3 mks)

f) Use the row reduction formula to find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 2 & 1 & 6 \end{bmatrix}$$

(8 mks)

g) Determine the basis of the matrix B below;

$$B = \begin{pmatrix} 1 & -3 & 2 \\ -2 & 6 & -4 \\ -1 & 3 & -2 \end{pmatrix}$$

(3marks)

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^T$ $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$-5 + (-10)(2)$

$30 - 27 = -3$

subspace.

Question TWO (20 Marks)

a) Given the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -3 & 1 & 2 \\ 3 & -2 & -1 \end{bmatrix}$$

Determine:

i) The determinant of A
ii) The matrix of the minors
iii) The adjoint of the co-factors of A
iv) Inverse of A

(12mks)

b) Determine whether the function $f(x)=x^2+4x+5$ is a linear combination of the functions $g(x)=x^2+x-1$ and $h(x)=x^2+2x+1$ (5marks)

c) Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 5 & 8 \end{bmatrix}$$

(3mks)

$-1 - 6 = -7$

QUESTION THREE (20 Marks)

a) Use the Cramer's rule to solve the following system of linear equations.

$$x + y + z = 6$$
$$2x + y = 4$$
$$2x + 3y + z = 11$$

(10 mks)

b) Use Gaussian elimination to solve the system of equations

$$2x - y + z \quad = 1$$
$$2x + 2y + 2z = 2$$
$$-2x + 4y + z = 5$$

(6mks)

c) Determine whether the set defined by the vector $(a, b, 2a + 3b)$ Is a subspace of $\mathbb{R}^3$

(4 mks)

QUESTION FOUR (20 Marks)

a) If $A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 5 & 6 & 7 & 2 \\ 8 & 9 & 10 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 5 & 6 \\ 8 & 9 \\ 3 & 7 \end{pmatrix}$, Find AB (5 mks)

b) Find the determinant of matrix below by reducing it first to an upper triangular matrix .

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 2 & 1 & -1 \end{pmatrix}$$

(5mks)

c) State (with brief explanation) whether the following statement is true or false. The vectors (1, 0, 0), (0, 2, 0), (1, 2, 0) span $\mathbb{R}^3$ (5 mks)

d) Determine whether the vectors (1, 2, 0), (0, 1, - 1), (1, 1, 2) are linearly independent in $\mathbb{R}^3$ (5 mks)

QUESTION FIVE (20 Marks)

a) Express $V = (1, -2, 5)$ in $\mathbb{R}^3$ as a linear combination of the vectors $\mathbf{u}_1 = (1, 1, 1)$, $\mathbf{u}_2 = (1, 2,3)$ and $\mathbf{u}_3 = (2, -1, 1)$ (6 mks)

b)
i) Define the basis of a vector space. (2 mks)
ii) Prove that the vectors (1, 1, 1), (0, 1, 2) and (3, 0, 1) form a basis for $\mathbb{R}^3$ (6 mks)

c)
i) Define linear transformation. (2 mks)
ii) Verify for the transformation defined by the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ that $A(V_1 + V_2) = AV_1 + AV_2$ (4 mks)