(Knowledge for Development)

KIBABII UNIVERSITY COLLEGE

A CONSTITUENT COLLEGE OFMASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2014/2015 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

COURSE CODE: MAT 204

COURSE TITLE: REAL ANALYSIS I

DATE: 29/4/15 **TIME:** 11.30AM -1.30PM

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INSTRUCTIONS TO CANDIDATES

Answer Question One in and Any other TWO Questions

TIME: 2 Hours

is a non-void subset of R which is bounded above then A has a LubA/SupA in R - Dually A is a non-void subset of R which is bounded below then infA/glbA exists in R -

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Show that if $x \neq 0$, then $x^{-1} \neq 0$ and x^{-1} is unique. (3mks)

b) For every $x \neq 0$, show that $x^2 > 0$, hence show that $1 > 0$. (3mks)

c) Let (S, <) be an ordered set and E a subset of S, if the least upper bound of E (lubE) and the greatest lower bound of E (glbE) exist. Show that i) the lubE is unique (4 mks)

ii) the glb.E is unique. (4 mks)

d) Show that $\sqrt{3}$ is an irrational number. (4mks)

e) State the completeness axiom for $\Re$ (2mks)

f) Let A be a nonvoid subset of $\Re$ which is bounded above. Define a set B by $B = \{-x; x \in A\}$, show that B is bounded below and -sup A= inf B. (4mks)

g) If a and b are given real numbers such that for every real number $\varepsilon > 0,\ a \leq b + \varepsilon$, show that $a \leq b$ (5mks)

h) Define an inductive set? (2mks)

QUESTION TWO (20 MARKS)

a) For any subset E of a metric space (X, ρ), prove that E^0 is an open set. (6mks)

b) Consider the metric space $(\Re, d)$ and let $f : \Re \to \Re$ be defined by $f(x) = |x|$. Show that f is uniformly continuous. (6mks)

c) Show that the limit of a convergent sequence is unique in a metric space (8mks)

QUESTION THREE (20 MARKS)

a) Show that every infinite set E contains a countable subset A. (7mks)

b) Differentiate between an algebraic and a transcendental number giving examples in each case (3mks)

By contradiction suppose $x^{-1} = 0$ then $x \cdot x^{-1} = x \cdot 0 = 0$ but $x \cdot x^{-1} = 1$
hence the supposition that $x^{-1} = 0$ is absurd and unacceptable, Thus $x^{-1} \neq 0$ as required. Let y and z be any two inverse of $x \Rightarrow xy = 1$ and $xz = 1 \Rightarrow xy = xz$ by cancellation law $y = z$ $\square$

c) Does the equation $x^2 + 1 = 0$ have a solution in $\Re$? Show your working. (4mks)

d) Define the following terms;

i. A metric space
ii. An interior point of a set E

(4mks)
(2mks)

QUESTION FOUR (20 MARKS)

a) Suppose that an open interval $(0,1)$ is equivalent to $\Re$. Show that $\Re$ is uncountable by alteration

(10mks)

b) State and provide a proof of Cauchy –Schwarz inequality. (10mks)

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QUESTION FIVE (20 MARKS)

a) Let A and B be nonvoid subsets of $\Re$ and define the set $A + B = \{x + y; x \in A, y \in B\}$,

b)

i. If A and B are bounded above, then show that A+B is also bounded above and sup(A+B)=sup.A+sup.B

ii. (5mks)

iii. If A and B are bounded below, then show that A+B is also bounded below and inf.(A+B)=inf.A+inf. B (5mks)

c) For every real numbers x and a, $a > 0$, show that $|x| \leq a$ iff $x \in [-a, a]$ (4mks)

d) Let A, B, C be nonvoid sets and $f : A \rightarrow B$ and $g : B \rightarrow C$ be bijections. Then, prove that $(g \circ f)^{-1}$ exists and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. (6 mks)

let $x \in \mathbb{R}$ since $(\mathbb{R}, <)$ is an ordered field if $x \neq 0$ then $x > 0$, $x > 0$ ~~x~~ $=$