(Knowledge for Development)

KIBABII UNIVERSITY COLLEGE

A CONSTITUENT COLLEGE OFMASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2014/2015 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 226

COURSE TITLE: CLASSICAL MECHANICS

DATE: 30/4/15 **TIME:** 11.30AM -1.30PM

INSTRUCTIONS TO CANDIDATES

Answer Question One in and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (COMPULSORY) (30MARKS)

(a) State Newton's laws of motion

(b) Define the following terms as used in classical Mechanics: (3marks)

(i) Kinematics

(ii) Statics

(iii) Torque

(iv) Force Field

(c) A particle of constant mass m moves in a space under the influence of a force of a field $\boldsymbol{F}$. Assuming that at times t_1 and t_2 the velocity is $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ respectively; prove that the work done is the change in kinetic energy. (4marks)

(d) Show that a force field $\boldsymbol{F}$ defined by (5marks)

$$\boldsymbol{F} = (y^2z^3 - 6xz^2)\boldsymbol{i} + 2xyz^3\boldsymbol{j} + (3xy^2z^2 - 6x^2z)\boldsymbol{k}$$ is a conservative force field. (5marks)

(e) (i) State the principle of conservation of angular momentum (1mark)

(ii) Is the force field given by $\boldsymbol{F} = x^2\, y\, z\, \boldsymbol{i} - xyz^2\boldsymbol{k}$ conservative? (4marks)

(f) A particle travels with uniform angular speed ω around a circle of radius b. Prove that its projection on a diameter oscillates with Simple Harmonic Motion of period $2\pi/\omega$ about the centre. (4marks)

(g) Solve the equation $d^2x/dt^2 + 2\,dx/dt + 5x = 0$ subject to the conditions $x = 5$, $dx/dt = -3$ at $t = 0$. (5marks)

QUESTION TWO (20MKS)

(a) A particle of mass m moves along a straight line under the influence of a constant force of magnitude F. If its initial speed is v_o, find,

(i) Speed

(ii) The velocity. (3marks)

(iii) The distance travelled after time t. (3marks)

(iv) Show that the speed of the particle at any position x is given by (3marks)

$$v = \sqrt{v_0^2 + (2F/m)x}$$

(3marks)

(b) An object of mass 20kg moves with simple harmonic motion on the $x-$ axis. Initially $(t=0)$ it is located at a distance 4 meters away from the origin $x=0$, and has velocity $15ms^{-1}$ and acceleration $100ms^{-2}$ directed towards $x=0$.

Find: (i) the position at any time.

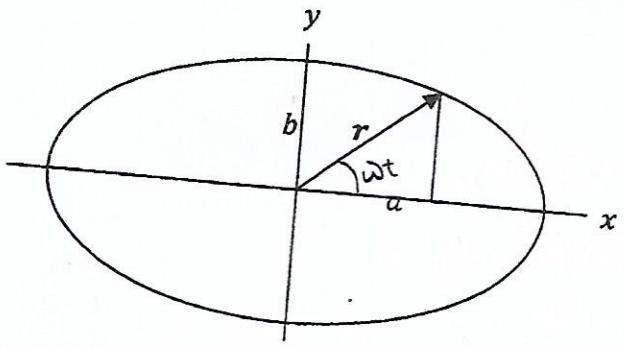
(ii) the amplitude, period and frequency of the oscillations.

(iii) the force on the object when $t = {}^{\pi}/_{10}\ s$.

QUESTION THREE (20 MARKS)

A particle of mass m moves in the xy plane so that its position vector is

$r = a\cos\omega t\, i + b\sin\omega t\, j$, where a, b and ω are positive constants and $a > b$.



(a) Show that the particle moves in an ellipse. (3marks)

(b) Show that the force acting on the particle is always directed toward the origin. (3marks)

(c) Show that the force field is conservative. (3marks)

(d) Find the potential energy at points A and B. (4marks)

(e) Find the work done in moving the force field from A to B. (3marks)

(f) Find the total energy of the particle and show that it is constant. (4marks)

QUESTION FOUR (20 MARKS)

A projectile is launched with initial speed v_0 at an angle α with the horizontal. Find:

(a) The position vector at any time. (5marks)

(b) The time it takes to reach the maximum height. (3marks)

(c) The maximum height reached. (3marks)

(d) The time of flight back to the earth and the range. (4marks)

(e) Show that the path of the projectile is a parabola. (3marks)

(f) Prove that the range of the projectile is a maximum when the launching angle (2marks)

(c) The maximum height reached.
(3marks)

(d) The time of flight back to the earth and the range. (4marks)

(e) Show that the path of the projectile is a parabola. (3marks)

(f) Prove that the range of the projectile is a maximum when the launching angle $\alpha = 45^{\circ}$.
(2marks)

QUESTION FIVE (20 MARKS)

(a) State Newton's law of Gravitation.
(2marks)

(b) Prove that if a particle moves in a central force field:

i) then its path must be a plane curve. (5marks)

ii) the momentum is conserved. (4marks)

(c) i) Show that $\mathbf{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative force field. (3marks)

ii) find the potential.
(4marks)

iii) find the work done in moving an object in this field from $(1, -2, 1)$ to $(3, 1, 4)$.
(2marks)

$W = \int_{t_1}^{t_2} F \cdot dr$

$P = m \cdot \frac{dv}{dt}$ and $\frac{dr}{dt} = v$

$\int_{t_1}^{t_2} m \frac{dv}{dt} \cdot v \cdot dt$

$\int_{t_1}^{t_2} mv\, dv \Rightarrow \frac{1}{2} mv^2 \Big|_{t_1}^{t_2}$

$\frac{1}{2} m(v^2(t_2) - v^2(t_1))$

$= W$

$F = ma$