STA 2495

W1-2-60-1-6

JOMO KENYATTA UNIVERSITY
OF
AGRICULTURE AND TECHNOLOGY

University Examinations 2017/2018

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

STA 2495 NON-LIFE INSURANCE MATHEMATICS

DATE: August 2018 **TIME: 2 HOURS**

***INSTRUCTIONS*: Answer question one and any other two questions**

Question one

a. Briefly discuss the non-life actuarial problems in practice. **(5 marks)**

b. Outline the components of the surplus process that determine the level of reserves in a general insurance at any given time. **(2 marks)**

c. Let X_i's, the claim amount of individual claims be iid exponential, and K, the number of claims be distributed as Poisson. Find the distribution of the total claims, Z. **(3 marks)**

d. The distribution of the number of claims from a non-life insurance business class is negative binomial with parameters $k = 4$ and $p = 0.9$. The claim size distribution is Pareto with parameters $\alpha = 5$ and $\lambda = 1{,}200$. Calculate the mean and variance of the aggregate claim distribution. **(4 marks)**

e. Discuss the significance of the Cramèr's inequality the risk analysis for a non-life insurance company. (4 marks)

f. For the Cramèr's inequality, show that $\Psi(x) \leq e^{-\kappa x}$ where κ is the solution of

$E\left[e^{-\kappa f}\right] = 1$. (4 marks)

g. Outline the pragmatic premium calculation principles (4 marks)

h. Compare the proportional reinsurance arrangements to the non-proportional reinsurance treaties. (4 marks)

Question two

While setting retentions in a non-life portfolio, assume claim numbers are distributed as Poisson with parameter λ. Let the retention be denoted by α, the need for reinsurance protection by q and the premium loading by δ.

a. Show that $q = \frac{E[Z]}{U}.v[Z].\left(-\frac{\ln \varepsilon}{2}\right)$ (7 marks)

b. For $\alpha = \frac{\delta}{q}$ discuss the practical rule of thumb of α

i. Under quota reinsurance (4 marks)
ii. Under surplus reinsurance (3 marks)
iii. Under excess of loss (3 marks)
iv. With stop loss → $\frac{\text{Net of claims}}{\text{Total gross [illegible]}}$ (3 marks)

Question three

a. If $X_1 \sim Poisson(\mu_1)$ and $X_2 \sim Poisson(\mu_2)$ are independent random variables, find the probability function of $Z = X_1 + X_2$ using convolutions. (5 marks)

b. Let K be the number of claims on a risk in one year. Suppose claims $\{X_1, X_2, \ldots\}$ are independent, identically distributed random variables, independent of K. Let Z be the total amount claimed.

i. Derive E(Z) and var(Z) in terms of the mean and variance of K and X_1 (4 marks)

ii. Derive an expression for the moment generating function $M_Z(t)$ of Z in terms of the moment generating functions $M_X(t)$ and $M_K(t)$ of X_1 and K respectively. (4 marks)

iii. If K has a Poisson distribution with mean λ, show that: (4 marks)

$$M_Z(t) = \exp\left(\lambda\left(M_X(t) - 1\right)\right)$$

iv. If K has a binomial distribution with parameters m and q, determine the moment generating function of Z in terms of m; q and $M_X(t)$. (3 marks)

Question four

a. Briefly discuss the commonly used continuous distributions for modeling individual claim sizes. (7 marks)

b. Using an example, discuss risk classification as used in experience rating. (5 marks)

c. Describe the application of credibility theory under experience rating in setting premiums of general insurance classes (8 marks)