

# STA 2401: TIME SERIES ANALYSIS CAT II TIME: 1 Hour

May 30, 2017

$$X_t = 0.25X_{t-1} + Z_t$$

*Handwritten notes:*  
 $X_t = 0.25X_{t-1} + Z_t$   
 $X_t = 0.25^2 X_{t-2} + 0.25Z_{t-1} + Z_t$   
 $X_t = 0.25^3 X_{t-3} + 0.25^2 Z_{t-2} + 0.25Z_{t-1} + Z_t$   
 $\vdots$   
 $X_t = 0.25^{t-1} X_1 + \sum_{j=0}^{t-2} 0.25^j Z_{t-j}$

- Investigate whether the following time series is stationary in weak sense  $X_t = \frac{1}{6}X_{t-1} + Z_t$  where  $\{Z_t\}$  is a white noise process with mean zero and variance 16.
- Differentiate between strictly stationary time series and weakly stationary time series.
- Define autoregressive and moving averages processes. Obtain the autocorrelation function of the processes:
  - (a)  $Y_t = Y_{t-1} - 0.5Y_{t-2} + Z_t$ , where  $\{Z_t\}$  is a purely random process.
  - (b)  $X_t = 1.6Z_{t-1} - 0.8Z_{t-2} + Z_t$  and  $\{Z_t\}$  is a white noise process with mean zero and variance 5
- Consider the process  $Y_t = Z_t + \frac{7}{10}Z_{t-1} - \frac{2}{10}Z_{t-2}$  where  $\{Z_t\}$  is a white noise. Investigate invertibility of the process.
- consider an AR(1) process given by  $X_t = 0.25X_{t-1} + Z_t$ . Use backward shift operator or otherwise to express this process as the sum of infinite moving average process. Hence obtain its Autocorrelation function.