STA 2492

JOMO KENYATTA UNIVERSITY

OF

AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2017/2018

FOURTH YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREES OF

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

STA 2492: CREDIBILITY THEORY AND LOSS MODELS

DATE: AUGUST 2018 TIME: 2 HOURS

Instructions: Attempt Question One and any Other Two Questions

QUESTION ONE (30 MARKS)

a) The following claims amounts are believed to come from a lognormal distribution with unknown parameters μ and σ^2.

50 87 103 119 126 154 183 207

i) Estimate the parameters μ and σ^2 using the method of moments. [2 marks]

ii) Estimate the parameters using the method of percentiles [3marks]

b) You are given the number of claims follows a negative binomial distribution with parameters r and $\beta = 3$. The claim severity has the following distribution

Claim size	Probability
1	0.4
10	0.4
100	0.2

The number of claims is impendent of severity of claims. Determine the expected number of claims needed for aggregate losses to be within 10% of expected aggregate losses with 95% probability. [4marks]

c) Let the Loss amounts be X and Let Y be the loss amounts after uniform inflation of r. Where $Y=(1+r)X$. For an ordinary deductible of d. Determine the expected cost per-loss. [5 marks]

d) The probability that an insured will have exactly one claim is θ. The prior distribution of θ has a probability density function

$$\pi(\theta)=\frac{3}{2}\sqrt{\theta} \qquad 0<\theta<1$$

A randomly selected insured is observed to have exactly one claim. Determine the posterior probability that θ is greater than 0.60. [4marks]

e) The number of claims by an individual insured in a year has a Poisson distribution with mean λ. The prior distribution of λ is gamma with parameter $\alpha=1$ and $\theta=1.2$. Three claims are observed in year 1 and no claims were observed in year 2. Using Buhlmann credibility, estimate the number of claims in year 3. [4 marks]

f) For an insurance company the number of losses per year has a Poisson distribution with $\lambda=10$. Loss amounts are uniformly distributed on $[0, 10]$. Loss amounts and the number of losses are mutually independent. There is an ordinary deductible of 4 per loss. Calculate the variance of aggregate payments in a year. [4marks]

g) Differentiate between policy deductible and a policy limit giving examples. [4marks]

QUESTION TWO (20 MARKS)

a) Losses follows a Pareto distribution with parameter $\alpha=2$ and $\theta=1{,}000$. For the coverage with policy limit 2,000 and after an inflation rate of 30%. Calculate the after inflation expected cost. [4 marks]

b) The number of claims, N made on an insurance portfolio follows the following distribution.

n	$\Pr(N=n)$
0	0.7
2	0.2
3	0.1

If a claim occurs, the benefit is 0 or 10 with probability 0.8 and 0.2 respectively. The number of claims and the benefit for each claim are independent. Calculate the probability that the aggregate benefit will exceed expected benefits by more than 2 standard deviations. [10marks]

c) The prior distribution of the parameter Θ has probability density function

$$\pi(\theta) = \frac{1}{\theta^2} \qquad 1 < \theta < \infty$$

Given $\Theta = \theta$, claim sizes follows a Pareto distribution with parameter $\alpha = 2$ and θ. A claim of three is observed. Calculate the posterior probability that Θ exceeds 2 [6 marks]

QUESTION THREE (20 MARKS)

a) For the aggregate loss distribution S.

i) The number of claims has a negative binomial distribution with $r = 16$ and $\beta = 6$

ii) The claim amounts are uniformly distributed on the interval $[0,8]$

iii) The number of claims and claim amounts are mutually independent.

Using the normal approximation for aggregate losses, calculate the premium such that the probability that aggregate losses will exceed the premium is 5% [5 marks]

b) The total claims for two policyholders are given below.

Policyholder	Year			
	1	2	3	4
X	730	800	650	700
Y	655	650	625	750

Using the Nonparametric Empirical Bayes method, determine the Buhlmann credibility premium for the policyholder Y. [6marks]

c) The number of claims has a Poisson distribution. The claim size has a Pareto distribution with parameters $\alpha = 0.5$ and $\theta = 6$ .the number of claims and claims sizes are independent. The observed pure premium should be within 2% of the expected pure premium 90% of the time. Determine the expected number of claims needed for full credibility [4 marks]

d) A portfolio consists of two types of policies .for type 1,the number of claims in a year has a Poisson distribution with 1.5 and the claims sizes are exponentially distributed with mean 5. For type 2, the number of claims in a year has a Poisson distribution with mean 2 and the claim sizes are exponentially distributed with mean 4.Let S be the total amount claimed on the whole portfolio in one year. All policies are assumed to be independent. Determine the mean and variance of S. [5marks]

Number of claims	Number of policy holders
0	2000
1	600
2	300
3	80
4	20
Total	3000