JOMO KENYATTA UNIVERSITY
OF
AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2018/2019
THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE/STATISTICS/BIOSTATISTICS
STA 2308: BAYESIAN INFERENCE I

DATE: APRIL 2019 **TIME: 2 HOURS**

INSTRUCTIONS: ***Answer Question 1 and attempt any other two Questions.***

Question 1 (30 marks)

a. An article describes a cancer testing scenario: 1% of women have breast cancer, 80% of mammograms detect breast cancer when it is there and 9.6% of mammograms detect breast cancer when it's not there. What are the chances of a woman having cancer? (i.e. probability of positive test result). (3 marks)

b. Let $X_1,\dots,X_n$ be iid Poisson(λ), and let λ have a gamma(α,β) prior distribution. Find the posterior distribution of λ. Show that the posterior mean approaches the maximum likelihood estimate as $n \to \infty$. (8 marks)

c. Briefly describe Gibbs sampler for parameters θ_1, θ_2, and θ_3, with joint distribution $p(y|\theta_1,\theta_2,\theta_3)$ p(y/σ (4 marks

d. Eric and Alex each suspect that a particular coin is biased and that the probability θ of "heads" is actually greater than 0.5. They propose two different experiments to test this belief statistically. Eric will toss the coin 10 times and record as his test statistic the number of heads obtained in the 10 trials. Alex will toss the coin repeatedly until 2 tails have been obtained in total. At this point Alex will use as his test statistic then number of heads he has obtained in his series of tosses.

It turns out that of Eric's 10 tosses, 8 were heads and at the point that Alex gets his second tail, he has accumulated 8 heads. Bayesian Betty decides to assume a (non-informative) prior for θ of

$$p(\theta) = 1, \quad 0 \leq \theta \leq 1.$$

Show that Bayesian inference based on the posterior distribution for θ will be exactly the same whether, as her observed data, she uses the results of Eric's experiment or uses the results of Alex's experiment. Discuss how this relates to the Likelihood Principle. (9 marks)

e. A drug company has developed n new drugs. Based on initial studies, the posterior probability that drug j is effective is $\theta_j \in (0,1)$, independent over j. The company must now decide which drugs are promising enough to conduct a large-scale trial to conclusively demonstrate their effectiveness. The company has unlimited money to initiate these trials. If drug j is really effective but they decide not to pursue a trial, it costs the company L_j shillings in lost profits. If the company pursues a trial for drug j and it turns out not to be effective, it costs the company K_j shillings.

What is the Bayes decision rule in this case? That is, a vector $d = (d_1, d_2, \ldots, d_n)$ where $d_j = 1$ if a trial is initiated for drug j and $d_j = 0$ otherwise. (6 marks)

Question 2 (20 marks)

a. Assume that you want to investigate the proportion (θ) of defective items manufactured at a production line. Your colleague takes at random sample of 30 items. Three were defective in the sample. Assuming a uniform prior for θ determine the posterior distribution. (10 marks)

b. A insurance company is faced with taking one of the following 3 actions: a_1: increase sales force by 10%; a_2: maintain present sales force; a_3: decrease sales force by 10%.

Depending on whether or not the economy is good (θ_1), mediocre (θ_2), or bad (θ_3), the company would expect to lose the following amounts of money in each case:

	a_1	a_2	a_3
θ_1	-10	-5	-3
θ_2	-5	-5	-2
θ_3	1	0	-1

The company believes that θ has the probability distribution $\pi(\theta_1) = 0.2, \pi(\theta_2) = 0.3, \pi(\theta_3) = 0.5$

(I) Order the action according to their Bayesian expected loss (equivalent to Bayes risk, here), and state the Bayes action. (7 marks)

(II) Order the actions according to the minimax principle and find the minimax action. (3 Marks)

Question 3 (20 marks)

a. Let x be a single observation from $N(\mu, 1)$, and let μ have a prior distribution

$$\pi(\mu) \propto e^{\mu}$$

On the whole real line.

(i) Find the posterior distribution of μ given x. (6 marks)

(ii) What is the Bayes estimator for μ under squared error loss. (6 marks)

b. Consider the maternal condition *placenta previa*, an unusual condition of pregnancy in which the placenta is implanted very low in the uterus, obstructing the fetus from a normal vaginal delivery.

An early study concerning the sex of *placenta previa* births found that of a total of 980 births, 437 were female. Assume a uniform prior distribution for the probability of a female birth.

(i) Construct the posterior distribution for the probability of a female birth given data. (6 marks)

(ii) Evaluate the posterior mean and posterior variance of the posterior distribution obtained in (i). (4 marks)

Question 4 (20 marks)

Sociologists have long been interested in *social mobility* – the transition of individuals between social classes defined on the basis of income or occupation. Consider a society with three social classes. Each individual may belong to the lower class (state 1), the middle class (state 2), or the upper class (state 3). suppose that *intergenerational mobility* is described by the transition matrix P

$$\begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.3 & 0.6 \end{pmatrix}$$

a. Determine the transition diagram from this transition matrix (5marks)

b. What will be the proportions of lower, middle and upper class after 5 years. (10 marks)

c. Sketch a R code for finding the long term trend of the transition matrix (5 marks)

2a)

$\theta \sim$ Binomial (θ)

$f(x|\theta) = \binom{n}{x} \theta^{x} (1-\theta)^{n-x}$

$L(x|\theta) = \theta^{\sum x_i} (1-\theta)^{n-\sum x_i}$

prior $p(\theta) = \{1$

<u>Posterior</u>

$p(\theta|y) = \theta^{\sum x_i} (1-\theta)^{n-\sum x_i}$

$= \theta^{(\sum x_i+1)-1} (1-\theta)^{(n-\sum x_i+1)-1}$

$= Beta(\sum x_i+1, n-\sum x_i+1) = \beta(\alpha^*, \beta^*)$

posterior mean $= \frac{\alpha^*}{\alpha^*+\beta^*} = \frac{\sum x_i+1}{\sum x_i+1+n-\sum x_i+1} = 4/32 = 0.125$