THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN TEACHER EDUCATION

MATHEMATICS

Paper 1

3 hours

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INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
- *Answer booklet*
- *Calculator and/or mathematical tables.*

*This paper consists of **FOUR** sections; A, B, C and D.*
*Answer **SIX** questions as follows:*

*Question **ONE** in Section A is **compulsory**.*
*Answer any **THREE** questions from Section **B**.*
*Answer any **ONE** question from Section **C**.*
*Answer any **ONE** question from section **D**.*

This paper consists of 5 printed pages.

Candidates should check the question paper to ensure that all
the pages are printed as indicated and that no questions are missing.

Turn over

SECTION A (25 marks)

Question one is compulsory

1. (a) Find the equation of the image of the line $y = 2x + 1$ under the transformation whose matrix is $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ (3 marks)

    (b) The distance from a point $P(x,y)$ to the point $A(6,0)$ is twice the distance from P to another point $B(0,3)$. Show that P lies on a circle. Hence, determine the centre and the radius of the circle. (5 marks)

    (c) Given the equation of an ellipse as $9x^2 + 16y^2 + 18x - 96y + 9 = 0$, determine the centre and vertices of the ellipse. (5 marks)

    (d) The position vectors of A and B are $\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ and $2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ respectively. Find:

    (i) the position vector of a point C that divides **AB** in the ratio -2:3; (2 marks)
    (ii) the co-ordinates of the mid-point of **AB**. (3 marks)

    (e) In a road test, 20% of the cars fail due to faulty steering while 30% fail due to faulty brakes. A car is allowed on the road only if it passes both tests. If a car is chosen at random, find the probability that it will be allowed on the road. (3 marks)

    (f) A right pyramid has a square base ABCD of length 10 cm and a vertex V. The slanting edge is of length 13 cm. Calculate the angle between the planes VAB and VCD

SECTION B: ANALYSIS AND CALCULUS (45 marks)

*Answer any **THREE** Questions from this section*

2. (a) Given that $f(x) = \frac{4 + 2x}{2 - 3x}$, show that $f^{-1}(x) = ff(x)$. (5 marks)

    (b) Using the substitution, $t = \tan\frac{1}{2}x$, find $\int \frac{dx}{1 + \cos x}$ (5 marks)

    (c) Calculate the area bounded by the curve $y = 6 \sin x$ and the line $y = 3$. (5 marks)

3. (a) Differentiate the following function with respect to x.
    $f(x) = \sin^{-1}x + \ln\sqrt{1 - x^2}$ (5 marks)

    (b) A particle starts from rest and accelerates at $(20 - 3t)\text{ms}^{-2}$

    (i) Find an expression of its velocity after t seconds. (4 marks)

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(ii)　Determine the maximum velocity of the particle.　(3 marks)

(iii)　Calculate the distance covered in the first 2 seconds.　(3 marks)

4.　(a)　Evaluate:

(i)　$\lim\limits_{x \to 0} \dfrac{\sqrt{x+4}-2}{x}$　(3 marks)

(ii)　$\lim\limits_{x \to \infty} \dfrac{2x^2+1}{6+x-3x^2}$　(3 marks)

(b)　Solve the differential equation.

$x - 2y\dfrac{dy}{dx} = \cos x$　(3 marks)

(c)　In a culture of yeast, the rate of fermentation is proportional to the amount of active ferment, Q. If the amount of active ferment doubles in 1 hour, find the amount of active ferment present after $2\frac{3}{4}$ hours.　(6 marks)

5.　(a)　Evaluate

(i)　$\displaystyle\int_4^9 \dfrac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$　**(5 marks)**

(ii)　$\displaystyle\int_1^3 \dfrac{x^2+x-6}{x(x^2-4)}\, dx$　**(5 marks)**

(b)　Find the equation of the tangent to the curve whose parametric equations are $x = \sqrt{t}$ and $y = t - \dfrac{1}{\sqrt{t}}$ at the point where $t = 4$ seconds.　(5 marks)

6.　(a)　Solve the differential equation

$\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = 4x^2 - 3x + 1$　(6 marks)

(b)　Gas is escaping from a spherical balloon at the rate of $900\ \text{cm}^3\text{s}^{-1}$. Determine how fast the surface area of the baloon is reducing when the radius is 360 cm.　(5 marks)

(c)　Given the curve $y = \dfrac{1}{2}x^{\frac{3}{2}}$, determine the length of the arc between $x = 0$ and $x = 2$.　(4 marks)

SECTION C - LINEAR ALGEBRA (15 marks)

Answer only ONE question from this section

7.   (a)   Given that

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} a & 3 \\ 3 & b \end{pmatrix} \text{ and } C = \begin{pmatrix} -4 & 7 & -7 \\ 3 & -5 & 5 \end{pmatrix}$$

determine the elements a and b given that, $BCA = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.   (5 marks)

(b)   Using the inverse matrix method, solve the simultaneous equations

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$

(7 marks)

(c)   Given the matrix $\begin{pmatrix} 1 & -2 & -4 \\ 2 & -5 & 3 \\ 1 & 6 & -3 \end{pmatrix}$, find its symmetric matrix.   (3 marks)

8   (a)   A matrix A satisfies the relation $A^2 + A - I = 0$, show that A^{-1} exists and $A^{-1} = A + I$ where **I** is the identity matrix.   **(4 marks)**

(b)   Find all the eigen values and the eigen vectors of the matrix $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{pmatrix}$

(11 marks)

SECTION D - NUMERICAL METHODS (15 marks)

*Answer only **ONE** question from this section*

9.   (a)   Find the positive root of the quadratic equation $3x^2 - 8x - 5 = 0$ correct to 5 decimal places using Newton - Raphson process. Take $x_0 = 3$ as a first approximation.

(6 marks)

    (b)   Use Simpson's rule with 5 ordinates to approximate $\int_0^1 e^{x^2}\, dx$ correct to 6 decimal places.

(5 marks)

    (c)   Find a linear expression in x which will act as an approximation to the function $f(x) = \sqrt[3]{x}$ in the neighbourhood of $x = 8$. Hence, find an approximate value of $\sqrt[3]{7.9}$.

(4 marks)

10.   (a)   The sides of a rectangle are measured as 10.2 cm and 5.74 cm. Find the percentage error in its area.

(4 marks)

    (b)   Obtain the Maclaurin's expansion for $\cos x$ up to the term in x^6. Use your results to find an approximation for $\cos(0.1)$ correct to 4 decimal places.

(6 marks)

    (c)   A table of values for a certain function is given below,

x	0.398	0.399	0.400
f(x)	0.408591	0.409671	0.410752

Find approximate values of $f''(0.398)$ using:

    (i)   linear interpolation;

(3 marks)

    (ii)   quadratic interpolation.

(2 marks)