9516/1
D.T.E.
MATHEMATICS
Paper 1
March/April 2010
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN TEACHER EDUCATION

MATHEMATICS

Paper 1

3 hours

Tutor Praise Joshua

INSTRUCTIONS TO CANDIDATES

1. *You should have the following for this examination:*
   - *Answer booklet*
   - *Calculator and/or mathematical tables*

2. *This paper consists of* ***FOUR*** *sections:* ***A, B, C*** *and* ***D***.

3. *Answer* ***SIX*** *questions as follows:*
   Question ***ONE*** *in Section* ***A*** *is* ***compulsory***.
   Answer any ***THREE*** *questions from Section* ***B***.
   Answer any ***ONE*** *question from Section* ***C***.
   Answer any ***ONE*** *question from section* ***D***.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

SECTION A: (25 marks)

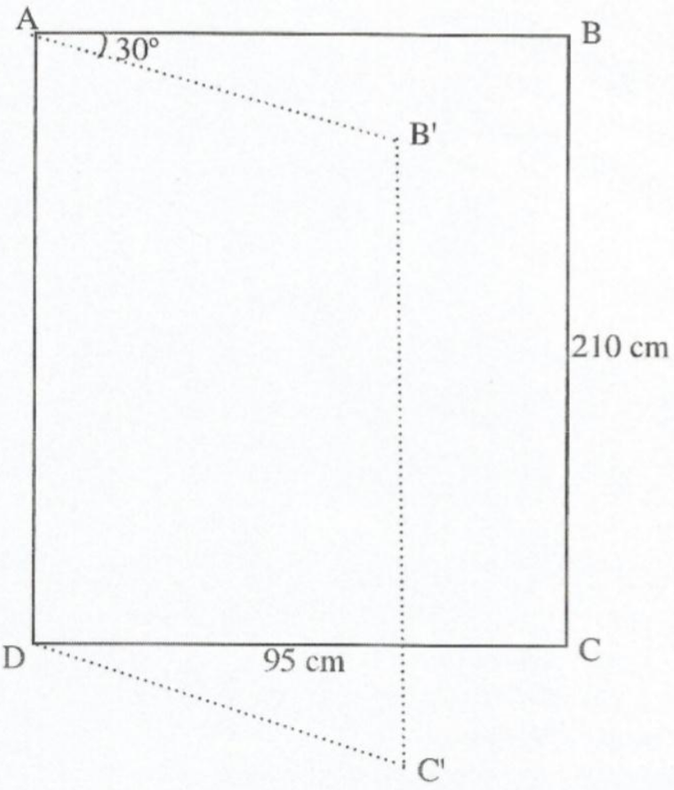
Question 1 is ***compulsory***.

1. (a) Determine the general matrix of a linear transformation which rotates a cartesian plane through an angle θ about the origin. (3 marks)

(b) Draw the curve $y^2 = x(x-2)$ for values of x: $-3 \leq x \leq 5$
Determine the equation(s) of the line(s) of symmetry. (5 marks)

(c) Find the length of the tangent to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$ from the point (5,7). (4 marks)

(d) A door, ABCD, 95 cm wide and 210 cm high is opened through an angle of $30°$ as shown in the figure below.



Calculate angle CAC'. (4 marks)

(e) In a triangle PQR, S and T are points on **PQ** and **QR** respectively such that **PS** = $\frac{2}{3}$ **PQ** and **QT** = $\frac{1}{3}$ **QR**. M is a point on **PT** such that **PM** = ¾**PT**. Taking **PQ** = **q** and **PR** = **r**, express **PT**, **RS** and **RM** in terms of **q** and **r**. Hence, show that R, M and S are collinear.

(4 marks)

(f) The heights of 40 boys measured in centimetres are shown in the table below.

Height (cm)	145-150	150-155	155-160	160-165	165-170	170-175	175-180	180-185
Frequency	1	1	4	15	12	3	3	1

(i) Draw a cumulative frequency curve for the data. (3 marks)

(ii) From the curve, determine the median height and the interquartile range. (2 marks)

SECTION B: CALCULUS & ANALYSIS (45 marks)

Answer any THREE questions from this section.

2\. (a) Evaluate the following:

(i) $$\lim_{x \to \infty} \frac{x^2 - 6x + 12}{5 + 2x^2}$$ **(4 marks)**

(ii) $$\lim_{x \to 0} \frac{6x - \sin 2x}{2x + 3\sin x}$$ (4 marks)

(b) Determine the equation of the tangent to the curve $x^3 + 3xy^2 + y^3 - 21 = 0$ at point (1, 2). (5 marks)

(c) The area of a circle is increasing at a rate of 5 cm^2s^{-1}. Find the rate of change of the circumference when the radius is 4 cm. (4 marks)

3\. (a) Show from first principles that the derivative of $\frac{1}{x}$ is $-\frac{1}{x^2}$. (4 marks)

(b) Differentiate with respect to x:

(i) $y = x^x$ (3 marks)

(ii) $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ (3 marks)

(c) Evaluate $\int_1^4 x^2 \ln x \, dx$ (5 marks)

4. (a) A closed oil can is made in the form of a right circular cylinder to contain one litre. If the amount of material to be used in making the can is a minimum, determine the dimensions of the can. (5 marks)

(b) A curve passing through point (1, -5) has a gradient function of $4x^3$. Find the value of x when y = 10. (3 marks)

(c) Show that $\int a^x dx = \frac{e^{x \ln a}}{\ln a} + C$ (3 marks)

(d) Determine the area bounded by the curve $y = x^2 - 2x$, the x-axis and the ordinates $x = -2$ and $x = 3$. (4 marks)

5. (a) Determine $\int \frac{x^3 + 5}{x^2 - 25} dx$ (5 marks)

(b) Solve the following differential equations:

(i) $\cos x \frac{dy}{dx} + 2y \sin x = \cos x + \cos^2 x$ (5 marks)

(ii) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$, given that $y = 3$ and $\frac{dy}{dx} = 2$ when $x = 0$. **(4 marks)**

6. (a) Find the volume of the solid generated when the area in the first quadrant of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is rotated through 360° about the x-axis. (5 marks)

(b) A car with initial velocity $2ms^{-1}$ moves in a straight line with acceleration, $a = \frac{5}{2} - \frac{t}{8} ms^{-2}$.

Calculate the distance travelled during the first four seconds. (5 marks)

(c) A radioactive substance decays at a rate proportional to its mass. The decay rate is 10 mg per week when its mass is 26 mg.

If after t weeks, the mass is M mg, find a formula for M in terms of t only.

(5 marks)

SECTION C - LINEAR ALGEBRA (15 marks)

*Answer any **ONE** question from this section.*

7. (a) Given the matrix $M = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 1 & 2 \\ 3 & 5 & 1 \end{pmatrix}$

obtain its inverse M^{-1}, using Cofactors. (7 marks)

(b) Solve the following set of simultaneous equations using Cramer's rule.

$$\begin{aligned} x + y + z &= 2 \\ x + 2y + z &= 6 \\ 6x + 5y + 4z &= 8 \end{aligned}$$

(8 marks)

8. (a) Given that $\begin{pmatrix} x & x \\ 3 & x-2 \end{pmatrix}$ is a singular matrix,

determine the values of x. (2 marks)

(b) Determine the value of λ for which the set of simultaneous equations below have a unique solution:

$$\begin{aligned} x + y + z &= 2 \\ x + 2y - 3z &= 5 \\ 2x + \lambda y - \lambda z &= 3\lambda \end{aligned}$$

(4 marks)

(c) Given the transformation defined by the matrix $\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$, find its eigen vectors. (8 marks)

SECTION D - NUMERICAL METHODS (15 marks)

*Answer any **ONE** question from this section*

9. (a) The following is an extract from a mathematical table.

X	9.000	9.001	9.002
X^3	729.0	729.2	729.5

Use either linear interpolation or extrapolation to estimate the value of:

(i) 9.0003^3 (2 marks)

(ii) 90.025^3 (3 marks)

(b) (i) Show that the cubic equation $x^3 + 3x - 15 = 0$ has only one real root which is in the neighbourhood of $x = 2$. (3 marks)

(ii) The cubic equation $x^3 + 3x - 15 = 0$ can be solved using the iterative formula $X_{n+1} = (15 - 3X_n)^{\frac{1}{3}}$.
Obtain the root correct to five decimal places using this iterative process. (3 marks)

(c) Using five ordinates, apply Simpson's rule to determine an approximate value of $\int_0^1 \frac{1}{1 + x^2}\, dx$, correct to four decimal places. (4 marks)

10. (a) Apply Trapezoidal rule to find, correct to eight decimal places, the approximate area of the region bounded by the curve $y = \frac{1}{\sqrt{3 + x^2}}$ between the ordinates $x = 1$ and $x = 4$, using six strips of equal width. (5 marks)

(b) Using Taylor's series expansion, expand $\cos(x - h)$ in ascending powers of h up to the term in h^6, hence determine cos 54.5° correct to four decimal places. (10 marks)