JOMO KENYATTA UNIVERSITY
OF
AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2019/2020 ACADEMIC YEAR.

Third Year Second Semester Examination for the Degree of Bachelor of Science in Actuarial Science, Biostatistics, Financial Engineering, Mathematics, Mathematics and Computer Science, and Statistics.

STA 2301: TESTS OF HYPOTHESIS

DATE: APRIL 2019 **TIME: 2 HOURS**

INSTRUCTIONS: Attempt Question ONE and any other TWO questions.

Question One (30 Marks)

(a) Given that $\bar{X} \geq 0.5$ is the critical region for the testing the null hypothesis that $H_0 : \theta = 0.3$ against the alternative that $H_1 : \theta = 0.25$ on the basis of a single observation from the population $f(x, \theta) = \frac{1}{\theta}e^{-\frac{1}{\theta}x}$ for $0 \leqslant X \leqslant \infty$. Compute

(i) pr(Type I error)

(ii) pr(Type II error) **(6 marks)**

(b) Let X be a normally distributed random variable with an unknown mean and a variance of 100. A random sample of size 34 is chosen form this population. If the critical region is given by $\omega = \{X : \bar{x} < 70\}$ and the hypothesis to be tested is; $H_0 : \mu = 85$ against $H_a : \mu < 85$. Determine;

(i) The power function and power of the test

(ii) The size of the test

(iii) The sample size that would make this test to be of size 0.06. **(12 marks)**

(c) Explain the steps involved in the Likelihood Ratio Test procedure and state the circumstance(s) under which it is preferred to the Neyman-Pearson approach.

(6 marks)

(d) A pharmaceutical company has installed a machine which fills automatically 5gms of drug in each phial. A random sample of 16 phials was taken and it was found to contain 5.08 gms on an average in a phial. The standard deviation of the sample was 0.12 gms. Test whether the machine is in order at 5% significance level. (6 marks)

Question TWO

The mean birth weight in JKUAT hospital is $\mu = 3315$ grammes, with standard deviation of $\sigma = 575$ grammes. Let X denote the birth weight in grammes in Juja hospital. If X is assumed to be normally distributed with mean μ and unknown variance σ^2,

(i) formulate a statistical hypothesis that would be tested to find out whether the mean birth weight in Juja is less than that of JKUAT hospital using a random sample of size $n = 30$.

(ii) Derive the Best Critical Region (BCR) for this test at $\alpha = 0.05$.

(iii) if this random sample of size $n = 30$ yielded $\bar{x} = 3189$ and $s = 488$, what conclusions do you make out of these?

Question THREE

(a) Explain the meaning of the following terms/phrases as used in statistical hypothesis testing

(i) a simple and composite hypothesis

(ii) a critical region

(iii) type I and type II errors (8 marks)

(b) Let X be the IQ scores for a certain population, and that X $N(\mu, 100)$. To test $H_0 : \mu = 110$ *vs* $H_1 : \mu > 110$, a random sample of size $n = 16$ from this population was taken. If a mean of $\bar{X} = 113.5$ was observed, do we accept or reject the null hypothesis at:

(i) 0.05 level of significance?

(ii) 0.1 level of significance?

(iii) what is the P-value of this test? (12 marks)

Question FOUR

Given a linear relationship:

$Y = \alpha + \beta x + \epsilon$ where α and β are unknown constants, and $\epsilon \asymp N(0, \sigma^2)$, consider testing the hypothesis $H_0 : \beta = 0$ against $H_1 : \beta \neq 0$. Show that

(i) $\hat{\alpha} = \bar{y} - \beta\bar{x}$, $\hat{\beta} = \frac{\sum x_i(y_i - \bar{y})}{\sum x_i(x_i - \bar{x})}$ and that $s^2 = \frac{(y_i - \hat{\alpha} - \hat{\beta} x_i)^2}{n}$ (4 marks)

(ii) $\hat{\alpha} \asymp N(\alpha, \frac{\sigma^2 \sum_{i=1}^{n} X_i}{n \sum_{i=1}^{n} (x_i - \bar{x})^2}$ (4 marks)

(iii) The likelihood ratio statistic corresponding to the above hypothesis is $\lambda = (1 - r^2)^{\frac{n}{2}}$ where r represents the correlation coefficient between X and Y. (4 marks)

(iv) Using the test statistic $T = \frac{\hat{\beta} \sum_{i=1}^{n} (x_i - \bar{x})^2 \, \frac{1}{2}}{\frac{n s^2}{n-2}}$, and the following summary statistics from the above model, $n = 10$, $\bar{x} = 1.4$, $\sum x_i^2 = 19.656$, $\bar{y} = 76.67$, $\sum y_i^2 = 58846.09$, $\sum x_i y_i = 1074.802$, show that the regression coefficient, β is significantly different from zero at $\alpha = 0.05$ level of significance. (8 marks)