a. Find the Fourier series expansion for $f(x) = x^2$. Hence use it to show that   [10marks]

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

b. By ratio test determine whether the following series converge $\sum_{n=1}^{\infty} \frac{(2n)!}{7^n (n!)^2}$ [5marks]

c. Given that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, use the limit comparison test to determine the convergence of $\sum_{n=1}^{\infty} \sin\frac{1}{n}$ [3marks]

d. If $a_n \nrightarrow 0$ as $n \rightarrow 0$ then $\sum_{n=1}^{\infty} a_n$ diverges. Prove   [4marks]

e. State and prove the Cauchy criterion for convergence   [8marks]