STA 2306W1-2-60-1-6

JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

University Examinations 2015/2016

THIRDYEAREXAMINATION FOR BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE/BIOSTATISTICS/FINANCIAL ENGINEERING/STATISTICS

STA 2306: REAL ANALYSIS FOR STATISTICS

DATE: DECEMBER 2015 **TIME 2 HOURS**

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY)

a) Find the radius and the interval of convergence of the power series [5 marks]

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n2^n}$$

b) Determine whether the sequence of functions $f_n(x) = \frac{nx}{1+n^2x^2}$ converges uniformly on $(0, \infty)$ [4 marks]

c) Use the integral test to determine the convergence of the series [5 marks]

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

d) Minimize $f(x, y) = x^2 + y^2$ subject to the constraint $2x + y = k$ [6 marks]

e) If $f(x, y) = \log(x^2 + y^3)$, find the mixed partials and the second order partial derivatives of this function [6 marks]

f) Use the Weierstrass M-test for convergence to show that the series

$$\sum_{n=1}^{\infty} n^2 e^{-nx}$$

converges uniformly on $[1, \infty)$ [4 marks]

$1 \cdot F(x, y, \lambda) = f(x, y) + \lambda g(x, y)$

QUESTION TWO

a) Prove that the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

converges if $p > 1$ and diverges if $p \leq 1$ [7 marks]

b) Find the minimum and maximum values of $f(x,y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$ [7 marks]

c) Find a formula for the nth partial sum S_n and evaluate the series, [6 marks]

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n-1)}$$

QUESTION THREE

a) Describe the integral test of convergence of a series [5 marks]

b) If $U = \log\sqrt{x^2 + y^2}$ show that $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$ [10 marks]

c) Use ratio test to determine the convergence of the series, [5 marks]

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

QUESTION FOUR

a) Find the Fourier series expansion for $f(x) = e^{-x}$ for $x \in (0, 2\pi)$ [10 marks]

b) Find the equation of the tangent plane to the surface $x^2 + xy^2 + z + 1 = 0$ at (2, -3, 4) [6 marks]

c) Use Weirstrass M-test to show that

$$\sum \frac{\sin^n x}{n^2 + x}$$

converges uniformly on $(0, \infty)$ [4 marks]