JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS FOR BACHELOR OF SCIENCE

CAT 1 FOR STA 2300 : Theory of Estimation

DATE. November 9, 2018 TIME: 1 HOUR

INSTRUCTIONS: Answer all Questions and in each case, clearly show your working.

Question One

Let $X_1, X_2, X_3 ... X_n$ be a random sample from a known PDF/PMF $f(x, \theta)$, where $\theta \in \Omega$ is unknown. Based on this scenario, define the following terms/Phrases as used in Statistical Inference. (Estimator, Estimate, Sufficient Statistic, Sampling Distribution, Unbiased Estimator) (5 MARKS)

A statistic $T = T(X)$ is a ~~suff~~ statistic for a parameter θ ~~if~~ for all sets A, ~~Pr~~ $(X \in A \mid T = t)$ are independent for all

Question Two

Let $X_1, X_2, X_3 ... X_n$ be iid $\sim N(\mu, \sigma^2)$, where both μ and σ^2 are unknown. Define the sample mean as $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Use $\bar{X}$ as an estimator μ to confirm that MSE consistency implies unbiasedness. (10 MARKS)

Question Three

Consider tennis players A and B who from time to time play a match against each other. Let us assume that the probability of A winning a set against B is p and the results of sets are independent (This is a somewhat oversimplified assumption, probability p may change over time, and results of sets within a match may be dependent. We will however take this assumption as a starting point for analysis.) Estimate the probability p, which reflects the relative strengths of players A and B. [5 marks]

$$0\left(\frac{2\theta}{3}\right) + \frac{\theta}{3} + \frac{4(1-\theta)}{3} + (1-\theta) = 1$$

Question Four

$$\theta = -\frac{1}{2}$$

Suppose that X is a discrete random variable with the following probability mass function.

X	0	1	2	3
P(X)	$2\theta/3$	$\theta/3$	$2(1-\theta)/3$	$(1-\theta)/3$

$$\frac{\theta}{3} + \frac{2(1-\theta)}{3} + 1-$$

where $0 \leq \theta \leq 1$ is a parameter. The following 10 independent observations (3,0,2,1,3,2,1,0,2,1) were taken from such a distribution. What is the maximum likelihood estimate of θ. (15 MARKS)