STA 2305/STA 2406

W1-2-60-1-6

JOMO KENYATTA UNIVERSITY

OF

AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2018/2019

THIRD/FOURTH YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE, BIOSTATISTICS, STATISTICS, FINANCIAL ENGINEERING AND OPERATIONS RESEARCH

STA 2305/STA 2406 STOCHASTIC PROCESSES

DATE: APRIL, 2019   TIME: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. *Answer question ONE and any other two questions*
2. *Be neat and show all workings*
3. *All questions except question one carry equal marks*

SECTION A

QUESTION ONE (30 MARKS)

a) Define the following terms
   - i) Stochastic matrix (2 Marks)
   - ii) Ergodic state (2 Marks)
   - iii) Irreducible Markov chain (2 Marks)

b) Let X be a random variable such that

$$\Pr\{X = k\} = p_k, \Pr\{X > k\} = q_k = \sum_{r=k+1}^{\infty} p_r, \, k \geq 0. \quad \text{If } P(s) = \sum_{k=0}^{\infty} p_k s^k \text{ and } Q(s) = \sum_{k=0}^{\infty} q_k s^k,$$

Show that $(1-s)Q(s)=1-P(s)$ and that $E(X)=Q(1)$. (7 Marks)

c) Given that Fibonacci numbers are given by $a_0 = 0, a_1 = 1, \quad a_n = a_{n-1} + a_{n-2}, \quad n \geq 2$. Find the generating function of the sequence $\{a_n\}$ (3 marks)

d) The probability generating function of a certain process is given by

$$G(s,t) = \frac{s e^{-\lambda t}}{\left[1 - \left(1 - e^{-\lambda t}\right)s\right]}$$

Determine the mean and variance of this process. (5 Marks)

e) Consider the three-state Markov chain having transitional probability matrix given by

$$\begin{bmatrix} 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Obtain the stationary distribution for this Markov chain. (6 Marks)

f) Given that the probability generating function of a process is $G(s) = \frac{1}{2} + \frac{1}{4}s + \frac{1}{4}s^2$ Find the probability of extinction of the process. (6 Marks)

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ii) With regard to a gamble with payoffs of -$5 000 and $12 000, Katy is indifferent between accepting $3 500 for certain and the gamble. What probability has Katy assigned to the event that the payoff will be $12 000?

QUESTION TWO (20 MARKS)

a) Cement Co is a company specializing in the manufacture of cement, a product used in the building industry. The company has found that when weather conditions are good, the demand for cement increases since more building work is able to take place. Cement Co is now trying to work out the level of cement production for the coming year in order to maximize profits. The company has received the following estimates about the probable weather conditions and corresponding demand levels for the coming year:

Weather	Probability	Demand
Good	0.25	350,000 bags
Average	0.45	280,000 bags
Poor	0.30	200,000 bags

Each bag of cement sells for $9 and costs $4 to make. If cement is unsold at the end of the year, it has to be disposed of at a cost of $0.50 per bag. Cement Co has decided to produce at one of the three levels of production to match forecast demand. It now has to decide which level of cement production to select. Construct a pay-off table to show all the possible profit outcomes and determine the level of cement production the company should choose, based on the conservative approach. **(5 marks)**

b) Burger Prince Restaurant is considering opening a new restaurant on Main Street. It has three different models, each with a different seating capacity. Burger Prince estimates that the average number of customers per hour will be 80, 100, or 120. The payoff table for the three models is on the next slide.

Model	Average no of Customers per hour		
	$S_1 = 80$	$S_2 = 100$	$S_3 = 120$
A	$10,0000	$15,0000	$14,0000
B	$8,0000	$18,0000	$12,0000
C	$6,0000	$16,0000	$21,0000
Prob	0.4	0.2	0.4

(i) Calculate the expected value for each decision. Which model is optimal? How much to the maximum is Burger Prince willing to pay in order to know the actual number of customer they expect to serve per hour? **(5 marks)**

(ii) Burger Prince must decide whether or not to purchase a marketing survey from Stanton Marketing for $500. The results of the survey are "favorable" or "unfavorable". The conditional probabilities are: P(favorable | 80 customers per hour) = 0.2, P(favorable | 100 customers per hour) = 0.5 and P(favorable | 120 customers per hour) = 0.9. Should Burger Prince have the survey performed by Stanton Marketing? What is the efficiency of this sample information? **(15 marks)**

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QUESTION FOUR (20 MARKS)

Consider the differential-difference equation of the zero-growth rate of birth-death process where $\lambda_n = n\lambda$, $\mu_n = n\mu$ and $\lambda = \mu$. Suppose the initial conditions are $p_n(0) = 1$ for $n = 1$ and 0 otherwise.

a) Obtain the distribution $p_n(t)$ of this process. (15 Marks)

b) Using the probability generating function of this process, determine the mean and variance of this process. (5 Marks)