$f(a) + f'(a)\,x-a + f''(a)\frac{(x-a)}{2!}$ $\frac{}{3!}$

$f(a)$ $f(a)=s$

$f(a)=1$

$s(1+4st$

$s(1$

W1-2-60-1-6

JOMO KENYATTA UNIVERSITY

OF

AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2016/2017

THIRD YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

STA 2391: STOCHASTIC PROCESSES FOR ACTUARIAL AND FINANCE

DATE: JUNE 2017 TIME: 2 HOURS

INSTRUCTIONS: Answer question ONE and any other TWO questions .

QUESTION ONE (30 MARKS)

(a) Differentiate the following terms:

(i) States of a process and state space of a process

(ii) Recurrent state and Transient state

(iii) Discrete time Stochastic process and Continuous time Stochastic process

[5 marks]

(b) Consider the Lower tail probabikity of a random variable X where $Pr(X = k) = p_k$ and $Pr(X \leq k) = q_k = p_k + p_{k-1} + ... + p_1 + p_0$ for $k \geq 0$. Show that the generating function of q_k is a function of the p.g.f of the random variable X.

[5 marks]

(c) Suppose $X_1, \quad X_2, ...X_N$ are i.i.d binomial random variables with parameter θ and N is a poisson random variable with parameter λ. Find the p.g.f of $S_N = X_1 + X_2 + ... + X_N$. Hence obtain the mean of S_N .

formula

[7 marks]

(d) Define the term "Generating function" of a random variable. Hence obtain the generating fiction of the sequence $\{n^2\}$ for $n = 0, \quad 1, \quad 2, \quad 3....$

[7 marks]

$H(s) = G[P(s)]$

$= e^{-\lambda(1-s)}$

$= e^{-\lambda(1-(q+ps)^n)}$

(e) Given the process $X(t) = B_1 + B_2 t$ where B_1 and B_2 are independent random variables with $E(B_i) = b_i$ and $Var(B_i) = \sigma_i^2$, $i = 1, 2$. Investigate stationarity of the process $X(t)$

[4 marks]

QUESTION TWO (20 MARKS)

Consider a pure birth process with difference differential equations

$p_n'(t) = -n\lambda p_n(t) + (n-1)\lambda p_{n-1}(t)$, $n \geq 1$ and $p_0'(t) = 0$, $n = 0$ with initial conditions $p_n(0) = 1$, $n = 1$ and $p_n(0) = 0$, $n \neq 1$.

(a) Obtain the solution of the process

(b) Use Feller's method to find the mean of the process.

[20 marks]

QUESTION THREE (20 MARKS)

(a) Define the following terms;

(i) Stochastic matrix

(ii) Persistent state A state is said to be persistent if $F_i = \sum f_{ii}^{(n)} = 1$

(iii) Irreducible Markov state

(iv) Ergodic state – A state that is persistent, non-null & aperiodic
Aperiodic –

[6 marks]

(b) Consider a Markov Chain with only four states $\{E_1, E_2, E_3, E_4\}$ and transition probability matrix

$$P = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{3}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

. Classify the states of this Markov Chain

[14 marks]

$(n-1)^2 + (n-1)$ $n^2 - n$
$n(n-1)$

QUESTION FOUR (20 MARKS)

(a) The probability generating function of a stochastic process is given by $G(s,t) = \frac{se^{-\lambda t}}{1-(1-e^{-\lambda t})s}$. Use the probability generating function technique to show that the mean and variance of the process are: $e^{\lambda t}$ and $e^{\lambda t}(e^{\lambda t} - 1)$ respectively.

[9 marks]

(b) Define Covariance stationarity of a stochastic process. Use the p.g.f technique to investigate covariance stationarity of the process $\{X_t\}_{t \in T}$ with probability distribution

$$P\{X_t = k\} = \begin{cases} \frac{e^{-\lambda t}(\lambda t)^k}{k!}; & k = 0, 1, 2, \ldots \\ 0; & elsewhere \end{cases}$$

[11 marks]