

### Expected Utility Questions

- a) ✓ An investor (Thomas) with wealth ( $w$ ) of £1000 is given an opportunity to take one of two prospects. The first prospect (A) offers a 25% chance of winning £200 and a 75% chance of winning £900. The second prospect (B) offers a 40% chance of winning £600 and a 60% chance of winning £800.
- Thomas' utility function is that of the natural log function [so  $U(w) = \ln(w)$ ]. Sketch Thomas's utility function and describe Thomas's attitude to risk and show how Thomas would evaluate the two prospects (A and B) above and choose between them.
  - Assuming that Thomas chose prospect (A), at what cash value would he be indifferent to taking the prospect or having the cash [i.e. what is the certainty equivalent of prospect (A)]?
  - A second investor (Sarah) has wealth of £750 but she has a utility function described as  $U(w) = \sqrt{w}$ . Explain whether Thomas or Sarah has a higher level of risk aversion. [Justify your answer and show your working clearly.]
- b) ✓ Consider a person with the following utility function over wealth:  $u(w) = \exp(w)$ , where  $e$  is the exponential function (and  $w$  = wealth in hundreds of thousands of dollars. Suppose that this person has a 40% chance of wealth of \$50,000 and a 60% chance of wealth of \$1,000,000 as summarized by  $P(0.40, \$50,000, \$1,000,000)$ .
- What is the expected value of wealth (in both \$s and utility)?
  - Construct a sketch graph of this utility function. Is this person risk averse, risk neutral, or a risk seeker?
  - What is this person's certainty equivalent for the prospect?
- ✓ An individual has the following utility function:  $u(w) = w^{0.5}$  where  $w$  = wealth.
- Using expected utility, order the following prospects in terms of preference, from the most to the least preferred:  $P_1(0.8, 1,000, 600)$   $P_2(0.7, 1,200, 600)$   $P_3(0.5, 2,000, 300)$
  - What is the certainty equivalent for prospect  $P_2$ ?
  - Without doing any calculations, would the certainty equivalent for prospect  $P_1$  be larger or smaller than that of  $P_2$ ? Why?
- ↳ Mary's total wealth is \$1000. She has been given a ticket to a fair lottery with pay offs of  $L(0.2, 20, 140)$ . Mary's utility function is  $u(w) = \exp(W/100)$ . Calculate how much a ticket to such a lottery would be worth to Mary. Use a sketch chart to illustrate your answer.
- ⊙ An investor is asked to compare two gambles. The first gamble offers £50 with a 40% chance and a 60% chance of receiving £500. The second an equal chance of receiving £100 or £500.
- How would an investor with a (natural) log utility function (so  $U(W) = \ln(W)$ ) evaluate the two gambles above and choose between them?
  - Describe the investors' attitude to risk?
  - How would an investor with an exponential utility function (so  $U(W) = \exp(W)$ ) evaluate these two gambles and choose between them? Describe the investor's attitude to risk.
- ⊙ A venture capitalist has £1 to invest in a new business. The business venture will either immediately repay nothing, £8.5 or £21 with equal probability. Assume the venture capitalist's utility function for evaluating risky outcomes is given by a quadratic equation of the form  $50 \times \text{payoff} - \text{payoff}^2$  (for payoff  $\leq$  £50). Draw a sketch of the venture capitalist's utility function for evaluating risky outcomes. Label all axes clearly. 2