STA 2301: TESTS OF HYPOTHESIS
Continuous Assessment Test II

INSTRUCTIONS: Attempt ALL questions.   TIME: 1 HOUR

a) Briefly explain the meaning of the following terms/phrases as used in statistical inference;

    (i)   Unbiased Estimator

    (ii)   Unbiased Test

    (iii) Consistent Test

    (iv) Power of a Statistical Test   [4 marks]

b) Let $X_1, X_2, ..., X_n$ be a random sample from the *PMF*

$$f(x; \theta) = \begin{cases} \theta^x (1-\theta)^{1-x}, & x = 0, 1 \\ 0, & otherwise \end{cases}$$

  i) Show that $T = \sum_{i=1}^{n} X_i$ has a *PMF* which belongs to a one-parameter exponential family of distributions. Hence find a complete sufficient statistic for θ.   [4 marks]

  ii) Find the Maximum Likelihood Estimator (MLE) of θ.   [4 marks]

  iii) Determine the Most Powerful (MP) size α test for the null hypothesis $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta = \theta_1$ (where $\theta_1 > \theta_0$). Hence deduce the Uniformly Most Powerful (UMP) size α test for H_0 against $H_1 : \theta > \theta_0$.   [8 marks]

c) Let $X_1, X_2, ..., X_n$ be a random sample from the Poisson distribution with the *PMF*

$$f(x; \theta) = \begin{cases} \dfrac{e^{-\theta} \theta^x}{x!}, & x = 0, 1, 2, ... \\ 0, & otherwise \end{cases} \quad (\theta > 0)$$

Find the Uniformly Most Powerful (UMP) unbiased size α test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$, where θ_0 is a specified number.

Give an expression for the power function.   [10 marks]