



W1-2-60-1-6

JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

University Examinations 2016/2017

**THIRD YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE**

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**THIRD YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF
BACHELOR OF SCIENCE IN FINANCIAL ENGINEERING**

STA 2309: RISK THEORY FOR ACTUARIAL SCIENCE

STA 2310: RISK THEORY FOR FINANCIAL ENGINEERING

DATE: JUNE 2017

TIME: 2 HOURS

INSTRUCTIONS: Answer question ONE and any other TWO questions

Question One (30 Marks)

- (a) Explain what you understand by the following terms [6 marks]
- i. False accept risk
 - ii. Risk management
 - iii. Risk aversion
- (b) Briefly discuss the application of payback and coefficient of variation as techniques of handling risk in capital budgeting. [6 marks]
- (c) A decision maker's utility function is given by $u(w) = \sqrt{w}$. The decision maker has wealth of $w = 10$ and faces a random loss X with a uniform distribution on $(0, 10)$. What is the maximum amount this decision maker will pay for complete insurance against the random loss? [5 marks]

- c) Consider the following cost payoff table (in \$1,000). Use the expected regret approach to determine the optimal decision. (5 marks)

Decision Alternative	State of Nature			
	S ₁	S ₂	S ₃	S ₄
D ₁	14	9	10	5
D ₂	11	10	8	7
D ₃	9	10	10	11
D ₄	8	10	11	13
Prior prob	0.5	0.2	0.2	0.1

- d) A patient enters the hospital with serious abdominal pains. Based on past experience Doctor Craig believes that there is a 28% chance that the patient has appendicitis and a 72% chance that the patient has nonspecific abdominal pains. Dr. Craig may operate on the patient now or wait 12 hours to gain more accurate diagnosis. In 12 hours Dr Craig will surely know whether the patient has appendicitis. The problem is that in the meantime, the patient's appendix may perforate (if he has appendicitis), thereby making the operation much more dangerous. Again based on past experience Doctor Craig believes that if he wait 12 hours there is a 6% chance that the patient will end up with a perforated Appendix, a 22% chance that the patient will end up with a normal appendicitis and a 72% chance that the patient will end up with nonspecific abdominal pains. From past experience Dr. Craig assesses the probabilities shown in the table below of the patients dying. Assume that Dr. Craig's goal is to maximize the probability that the patient will survive. Use a decision tree to help Dr. Craig make the right decision. (7 marks)

Situation	Probability that patient will die
Operation on patient with appendicitis	0.0009
Operation on patient with nonspecific abdominal pains	0.0004
Operation on perforated appendixes	0.0064
No Operation on patient with nonspecific abdominal pains	0

- e) Katy Chan has inherited \$100,000 from her aunt. She wants to invest the money for one year after which she will go on a trip to Egypt with the return on investment (i.e., she wants to save the \$100,000 and spend the return). She wants to maximize the return on her one-year investment. Katy is considering the following options:
- Option A - Buy bonds at a guaranteed interest rate of 12%.
 - Option B - Invest in a new computer company. If the company is successful she will get a 20% return on her investment whereas if it fails she would lose 5% on her investment. The probability that the computer company will be successful is estimated at 80%.
 - Option C - Invest half the money in bonds and half in the computer company. Profit or loss will be in proportion to the amount of money invested.
- Suppose that Katy has the following utilities: $U(20,000) = 1$, $U(16,000) = 0.9$, $U(12,000) = 0.8$, $U(3,500) = 0.6$ and $U(-5,000) = 0$.
- i) Using expected utility maximization criterion, determine Katy's optimal decision.

QUESTION THREE (20 MARKS)

Nomura Inc. has a contract with one of its customers, Mackenzie Tar & Grease to supply a unique liquid chemical product that is used by Mackenzie in the manufacture of a lubricant for aircraft engines. Because of the chemical process used by Nomura, batch size for the liquid chemical product must be 1000 pounds.

Mackenzie has agreed to adjust manufacturing to the full batch quantities, and will order either one, two, or three batches every three months. Since an aging process of one month is necessary for the product, Nomura will have to make its production (how much to make) decision before Mackenzie places an order. Thus, Nomura can list the product demand alternatives of 1000, 2000, or 3000 pounds, but the exact demand is unknown. Unfortunately, Nomura's product cannot be stored more than two months without degenerating into a viscous and highly corrosive toxic compound, linked in laboratory tests to the Ebola virus.

Nomura's manufacturing costs are \$150 per pound, and the product sells at the fixed contract price of \$200 per pound. If Mackenzie orders more than Nomura has produced, Nomura has agreed to absorb the added cost of filling the order by purchasing a higher quality substitute product from Miyamoto, another chemical firm. The substitute Miyamoto product, including transportation expenses, will cost Nomura \$240 per pound. Since the product cannot be stored more than two months, Nomura cannot inventory excess production until Mackenzie's next three-month order. Therefore, if Mackenzie's current order is less than Nomura has produced, the excess production will be reprocessed and valued at \$50 per pound.

The inventory decision in this problem is how much Nomura should produce given the costs and the possible demands of 1000, 2000, and 3000 pounds. From historical data and an analysis of Mackenzie's future demands, Nomura has assessed the probability distribution for demand shown in the table below

Demand	Probability
1000	0.30
2000	0.50
3000	0.20

- (a) Develop a payoff table for the Nomura problem
- (b) How many batches should Nomura produce every three months? Use the expected monetary value criterion
- (c) How much of a discount should Nomura be willing to allow Mackenzie for specifying in advance exactly how many batches will be purchased?

Nomura has detected a pattern in the demand for the product based on Mackenzie's previous order quantity. Let

I_1 = Mackenzie's last order was 1000 pounds

I_2 = Mackenzie's last order was 2000 pounds

I_3 = Mackenzie's last order was 3000 pounds

The conditional probabilities are as follows:

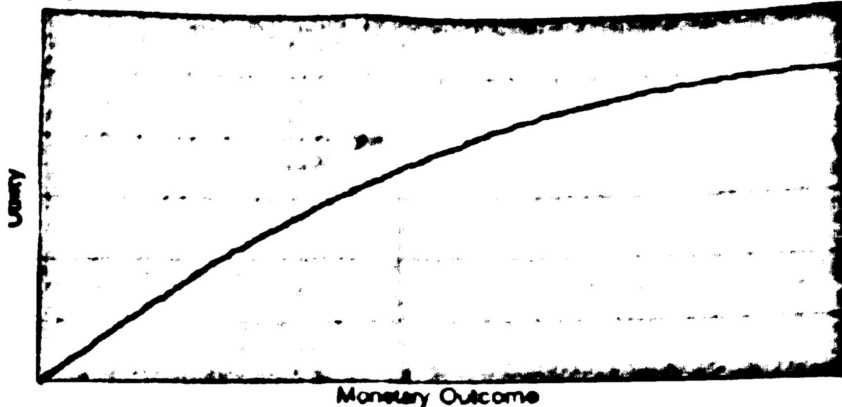
$$\begin{array}{lll}
 P(I_1|s_1) = 0.10 & P(I_2|s_1) = 0.40 & P(I_3|s_1) = 0.50 \\
 P(I_1|s_2) = 0.22 & P(I_2|s_2) = 0.68 & P(I_3|s_2) = 0.10
 \end{array}$$

$$P(I_1|s_1) = 0.80 \quad P(I_2|s_1) = 0.20 \quad P(I_1|s_2) = 0.00$$

- (d) Develop an optimal decision strategy for Nomura.
 (e) What is the expected value of sample information?
 (f) What is the efficiency of the information for the most recent order?

QUESTION FOUR (20 MARKS)

- a) Define the term utility and distinguish between a risk averter and a risk seeker. (3 marks)
 b) A decision maker's utility function is graphed below what type of decision maker does it represent and why? (1 mark)



- c) Damson has a 40% chance of earning \$2500/month and a 60% chance of earning \$1600/month. If his utility function is $u(w) = w^k$
- Verify that $u(w) = w^k$ represents a risk averter's utility function. (3 marks)
 - Find the expected earnings, the expected utility and the certainty equivalence of this Damson's gambling option. (3 marks)
- d) Joan's utility function for her asset position is given by $u(w) = w^k$. Currently Joan's assets consist of \$10,000 in cash and \$90,000 home. During a given year there is 0.001 chance that Joan's home will be destroyed by fire or other causes. How much would Joan be willing to pay for the insurance policy that would replace her home if it was destroyed? Hint: first find the ending wealth for all possible outcomes (7 marks)
- e) Compute the risk premium for I_2 in part d above [$RP(I_2) = EV(I_2) - CE(I_2)$]. In total how much would Joan be willing to pay to avoid the risk of her home being destroyed? (4 marks)