JOMO KENYATTA UNIVERSITY
OF
AGRICULTURE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS 2014/2015
THIRD YEAR SUPPLEMENTARY/SPECIAL EXAMINATIONS FOR THE DEGREE OF
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE
STA 2301: TESTS OF HYPOTHOTHESIS

DATE: MARCH 2015 TIME: 2 HOURS

INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS.

QUESTION ONE (30 MARKS)

(a) Define the following terms: [6 marks]

(i) A statistical hypothesis.
(ii) A test of a statistical hypothesis.
(iii) Type I error.
(iv) Power of a test.
(v) Critical region.
(vi) Size of a test.

(b) Suppose a random variable X follows the normal distribution with unknown mean u and variance $6^2 = 9$. A random sample of size n = 16 is taken on X. To test the null hypothesis H^0: u = 2 against the alternative H_1: u = 4.47, the critical region is given as $C = \{(x_1, x_2, \ldots, x_n): \bar{x} > 3.23\}$.

Compute:

(i) Pr (Type I error)
(ii) Pr (Type II error)
(iii) The power of the test at $\mu = 4.47$ [7 marks]

(c) State without proof, the Neyman-Pearson Lemma, giving precisely what kind of hypothesis applies. [5 marks]

(d) (i) Define the term probability value (P-value) and explain briefly how it can be used in testing a statistical hypothesis. [2 marks]

(ii) An investor wants to determine whether the mean income of workers working in a town two miles from a proposed estate building site exceeds \$24,400. What conclusion can be made at 5% level of significance if the mean income of a random sample of 60 workers working in that town is \$24,524 and standard deviation of \$763. Use P-value to answer this question. [5 marks]

(e) State and explain briefly the various steps involved in testing a hypothesis. [5 marks]

QUESTION TWO (20 MARKS)

(a) Let $x_1, x_2, \ldots, x_n$ be a random sample of size $n = 100$ from the normal distribution $N(\mu, \sigma^2)$ where $\sigma^2 = 400$. A size $\alpha = 0.05$ test for H_0: $\mu = 165$ against H_1: $\mu = 168$ is given by a Best Critical Region (BCR) $C = \{(x_1, x_2, \ldots, x_n): \bar{x} > k\}$

(i) Determine the value of K. [5 marks]

(ii) Give the expression for the power function, hence compute the power of the test when $\mu = 172.22$. [6 marks]

(iii) Give a uniformly most powerful (UMP) test for H_0: $\mu = 165$ against H_1: $\mu > 165$. Explain how this is derived from the Best Critical Region. [5 marks]

(b) Given that N follows a Poisson(λ) distribution and X_i's are iid standard normal variables

(i) Find the mgf of $S = X_1 + X_2 + \cdots + X_N$. [5 marks]

(ii) Using the mgf calculated in part (b)(i) above, confirm that [5 marks]

$$E[S] = E[X]E[N]$$
$$Var[S] = E[N]Var(X) + (E[X])^2 Var(N)$$

QUESTION THREE (20 MARKS)

(a) Determine the adjustment coefficient if the claim amount is exponential with parameter $\beta > 0$. [5 marks]

and the no of claims follow poisson(λ)

(b) Ronaldo a famous footballer has taken out special insurance which pays him some money if he is injured. If the injury is so severe that he can no longer play football, then the insurance will pay him \$1B and the policy is terminated. However, if he is injured but he manages to recover, then the insurance will pay him \$0.1B and the policy continues. The insurance company believes that the probability of an injury is 0.2 and that the probability of more than one injury in a year is 0. However, if Ronaldo is injured, then there is a 75% chance that he will recover. Assuming that premiums are paid in advance, and that the insurer pays claims at the end of the year that this is the only policy that the insurance company writes. Calculate:

(i) The annual premium charged assuming that the company uses a premium loading of 30%. [6 marks]

(ii) The probability that ruin occurs in the first 2 years if the initial reserve is \$0.1B. [9 marks]

QUESTION FOUR (20 MARKS)

(a) An insurer undertakes a risk X after collecting the premium. He owns a capital of $\omega = 100$. What is the maximum premium the insurer is willing to pay to a reinsurer to take over complete risk if his utility function is $u(\omega) = \omega^{\frac{1}{2}}$ and that $Pr(X = 0) = Pr(X = 36) = 0.5$. Find

(i) The exact premium. [5 marks]

(ii) The approximate premium. [5 marks]

(b) A general insurance company is planning to set up a new class of travel insurance. It plans to start the business with 2 million and expects claims to occur according to a Poisson process with parameter 50. Individuals claims are thought to have a gamma distribution with parameters $\alpha = 150$ and $\beta = 4$. A premium loading factor of 30% is applied. Explain how each of the following changes to the company's model will affect the probability of ultimate ruin.

(i) A 28% premium loading factor is applied instead. [6 marks]

(ii) The individual claims are found to have a gamma distribution with parameters $\alpha = 150$ and $\beta = 2$. [2 marks]

(iii) The Poisson parameter is now believed to be 60. [2 marks]

THE END