UNIVERSITY OF EMBU

2018/2019 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

STA 406: APPLIED STOCHASTIC PROCESSES

DATE: APRIL 11, 2019 **TIME: 8:30 AM – 10:30 AM**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) By use of an appropriate example, describe a stochastic process (4 marks)

b) Define

i) Branching process (2 marks)

ii) Generation (2 marks)

iii) Ancestor (2 marks)

c) Obtain the probability generating function hence determine the mean and variance of a Poisson process (5 marks)

d) Outline the steps of obtaining P^n in Markov chain transition probability matrix. (5 marks)

e) A certain Kind of nuclear particles split into 0, 1 or 2 particles with probability ¼, ½ and ¼ respectively and then dies. The individual particles are independent of each other. Given a particle, let Z_1, Z_2 and Z_3 denote the number of particles in the 1st, 2nd and 3rd generations respectively. Find $P(Z_2 > 0)$ (5 marks)

f) Suppose that the p.g.f of the first generation of a process is $G(s) = p_0 + p_1 s + p_2 s^2$ where $p_0 = \frac{1}{12}$, $p_1 = \frac{2}{3}$ and $p_2 = \frac{1}{4}$. Find the probability of extinction (5 marks)

QUESTION TWO (20 MARKS)

a) Write down the difference differential equation of a Simple birth process (2 marks)

b) Show that the Lagrange's linear equation of the process given above is (5 marks)

$$\frac{\partial G(s,t)}{\partial t} + \lambda s(1-s)\frac{\partial G(s,t)}{\partial s} = 0$$

c) Given that the initial condition $P_n(0) = \begin{cases} 1, & n = 1 \\ 0, & otherwise \end{cases}$, show that the probability that a population is of size n at time t is $P_n(t) = e^{-\lambda t}\left(1 - e^{-\lambda t}\right)^{n-1}$ (7 marks)

d) Show that the mean and variance of the process is $e^{\lambda t}$ and $e^{\lambda t}(e^{\lambda t} - 1)$ respectively (6 marks)

QUESTION THREE (20 MARKS)

a) Show that for n^th generation in a branching process the compounding pgf is given by $G_n(s) = G_{n-1}(G(s))$ (5 marks)

b) For the process given in (a) above, determine

i) Mean of the n^th generation (5 marks)

ii) $\text{Var}(Z_n)$ (8 marks)

Given that $\mu = E(Z_1)$ and $\sigma^2 = var(Z_1)$

c) Given any two applications of branching process (2 marks)

QUESTION FOUR (20 MARKS)

The difference differential equation of a simple birth-death process is given by

$$P_n'(t) = -(\lambda + \mu)nP_n(t) + \lambda(n-1)P_{n-1}(t) + \mu(n+1)P_{n+1}(t), \qquad n \geq 1$$

$$P_0(t) = \mu P_1(t), \qquad n = 0$$

Given the initial condition $P_n(0) = \begin{cases} 1, & n = 1 \\ 0, & otherwise \end{cases}$, use Feller's method to obtain

a) Mean (10 marks)

b) Variance (10 marks)

QUESTION FIVE (20 MARKS)

Consider the queuing process whose difference differential equations are

$$P_n'(t) = -(\lambda + n\mu)P_n(t) + \lambda P_{n-1}(t) + (n+1)\mu P_{n+1}(t), \qquad n \geq 1$$

And

$$P_n'(t) = -\lambda P_0(t) + \mu P_1(t), \qquad n = 0$$

Suppose that the initial condition is $P_n(0) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$, use probability generating function technique to solve for $P_n(t)$. Hence deduce the mean and variance of the process

(20 marks)

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