

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE OF
BACHELOR OF EDUCATION ARTS

MATH 123/113: VECTORS AND MECHANICS

STREAMS: BED (ARTS)

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 25/07/2018

5.30 PM – 8.30 PM

INSTRUCTIONS:

- All Questions are Compulsory
- Do not write on the question paper

Question One (30 Marks)

- (a) Define the following terms
- Mechanics
 - Vector
 - Moment

[3 marks]

- (b) Calculate the angle between the vectors

$$\tilde{a} = 4\tilde{i} - 2\tilde{j} + 1\tilde{h}$$

$$\tilde{b} = 2\tilde{i} + 5\tilde{j} - 7\tilde{k}$$

- (c) Given $\tilde{a} \cdot \tilde{b} = \tilde{a} \cdot \tilde{c}$, show that \tilde{a} is perpendicular to $\tilde{b} - \tilde{c}$

[4 marks]

- (d) Given $\vec{AB} = \tilde{a}$ and $\vec{AC} = \tilde{b}$, show that the area of triangle ABC is given by Area

$$\frac{1}{2} \sqrt{(ab)^2 - (\tilde{a} \cdot \tilde{b})^2}$$

[4 marks]

- (e) Determine the co-ordinate of the point where the line $\tilde{r} = \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$ meets the plane

$$\tilde{r} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 12 \quad [5 \text{ marks}]$$

- (f) Calculate the work done when a force $\vec{F} = -3\hat{i} + 4\hat{j} - 5\hat{k}$ moves through a displacement $\tilde{d} = -\hat{i} + 3\hat{j} - \hat{k}$ [5 marks]

- (g) A projectile is launched at an angle θ to the horizontal with an initial velocity v_0 m/s. show that its maximum range is given by $R_{max} = \frac{v_0^2}{g}$ [5 marks]

Question Two (20 Marks)

- (a) A force whose point of application is $(1, -2, 3)$ is given by $\vec{F} = 2\hat{i} + 3\hat{j} - 4\hat{k}$. Calculate the magnitude of the moment of the force about the point $(1, 2, -3)$ [5 marks]

- (b) Given $\vec{F} = \sin 2x \hat{i} - 3 \cos y \hat{j} + \tan z \hat{k}$, determine $\text{Div } F$ at $(\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4})$ [4 marks]

- (c) Show that the magnitude of vector product of two vectors $\tilde{a} \wedge \tilde{b}$ is equal to the area of the parallelogram where $\tilde{a} \wedge \tilde{b}$ form adjacent sides. [5 marks]

- (d) If $\tilde{a} \wedge \tilde{b}$ are vectors and $m > 0$ is a scalar, show that vector addition is distributive. [5 marks]

Question Three (20 Marks)

- (a) State Lami's theorem. [2 marks]

- (b) A body of weight 50N hang from a string attached to a point on a vertical wall. The string will break if its tension exceeds 60N. if the body is pulled away from the wall by a horizontal force PN, what angle does the string make with the vertical when it breaks and what is the value of P? [6 marks]

- (c) A particle moving in a straight line with constant acceleration travels 10m in the first second and 15M in the second second. Determine

- (i) Its initial velocity [2 marks]
- (ii) Its acceleration [2 marks]
- (iii) The distance it travels during the third second. [3 marks]

- (d) Calculate the volume of the parallelepiped defined by vectors

$$2\hat{i}-3\tilde{j}+4\tilde{k}, \hat{i}-2\tilde{j}+\tilde{k} \wedge 4\hat{i}+\tilde{j}-3\tilde{k}$$

[5 marks]