MACHAKOS UNIVERSITY

2018/2019

SMA 301: REAL ANALYSIS II - CAT I

Instructions: Answer ALL questions. [30 Marks]

- (1) If S is a finite set we sometimes write #S to denote the number of elements in S.
- Let $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$. For any $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ in \mathbb{R}^n define

$$d(x,y) = \#\{i : x_i \neq y_i\}$$

That is d(x, y) are the number of components of x and y that are different componentwise. For instance if $x = (2, 3.1, \frac{1}{2})$ and $y = (2, \frac{1}{3}, \frac{1}{2})$ then d(x, y) = 1. Prove that d is a metric on \mathbb{R}^3 . [8 Marks]

- (2) Let X be a set and suppose that d_1 and d_2 are metrics on X. [8 Marks]
 - (i) Show that the function $d: X \times X \to \mathbb{R}$ defined by

$$d(x,y) = d_1(x,y) + d_2(x,y) \text{ for all } x, y \in X$$

is also a metric on X.

(ii) Let d_1 and d_{∞} be the metrics defined on \mathbb{R}^2 by

$$d_1(x, y) = |x_1 - y_1| + |x_2 - y_2|$$

$$d_{\infty}(x, y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$$

where $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are elements of \mathbb{R}^2 . Calculate the distance

d(p,q)

where $p = (-1, -4), q = (6, 7) \in \mathbb{R}^2$ and where $d = d_1 + d_{\infty}$.

- (3) Let d denote the usual metric on \mathbb{R} and let A be the set $A = [4, 5] \cup (6, 7) \subset \mathbb{R}$. Is A and closed set in (\mathbb{R}, d) ? Justify your answer. [4 Marks]
- (4) Let (X, d) be a metric space and $A \subset X$ be a subset. [5 Marks]
 - (i) What do we mean by the term *limit points* of A?
 - (ii) Suppose $X = \mathbb{R}^2$ and that d is the Euclidean metric on \mathbb{R}^2 . Let A be the set

$$A = \{ x = (x_1, x_2) \in \mathbb{R}^2 : 3 \le x_1 \le 4, 2 \le x_2 < 4 \}.$$

What are the limit points of A?

- (5) Let (X, d) be a metric space and $A, B \subset X$ be subsets of X. [5 Marks]
 - (a) What do we mean by the term closure of A?
 - (b) Show that $\overline{A} \cup \overline{B} = \overline{A \cup B}$.