**MACHAKOS UNIVERSITY**

**SMA 300: REAL ANALYSIS I**

**CAT 1(25** $×1.2$**) marks**

1. Give definitions of each of the following .In each case give an example.
2. Rational number. (2 marks)
3. Neighborhood of a point. (2 marks)
4. Open set . (2marks)
5. Limit point. (2marks)
6. Closed set (2marks)
7. For each of the following sets determine the largest $ε$ such that it contains an $ε$- neighborhood about $x\_{0}$ .
8. $M=\left( \frac{3}{4} , 1 \frac{3}{5} \right], x\_{0}=1\frac{1}{2}$ (ii) $N=(-\infty , 2.3)$, $x\_{0}= -\frac{2}{5}$ . (iii) $T=\left\{x:x^{2}\leq \frac{9}{16}\right\}, $ $x\_{0}=\frac{4}{7}$ (6marks)
9. Prove that if $ P$ and $Q$ are neighborhoods of a point $x$, then $P∩Q $is also a neighborhood of the point $x$. (5marks)
10. Prove that the square of an odd integer is odd. (4marks)

**CAT 2 (25** $×1.2$**) marks**

1. The following are subsets of $R$:
2. $A=(-3\frac{1}{5} , 1]$ (ii) $B=\{\frac{\left(-1\right)^{n} \left(n+1 \right)}{n+2} :n \in N\}$ (iii) $C=\left\{x \in Q :x^{2}\leq 7\right\}.$

 For each set determine the Supremum, Infimum, Maximum value and Minimum value

 (7marks)

1. Define the concept of a closed subset of $X$. (1mark)
2. If $F\_{1}, F\_{2}…$ $F\_{n}$ are closed subsets of $X$ prove that $\bigcap\_{i=1}^{n}F\_{i} $is a closed subset of $X$ i.e any finite collection of closed sets is closed . (4marks)
3. Let $F\_{n}=\left(1-\frac{1}{n} , 3+\frac{1}{n}\right)$ , $F\_{n}⊂R,$ $n\geq 1$. Find:
4. $\bigcap\_{i=1}^{4}F\_{i}$ 2. $\bigcap\_{i=1}^{\infty }F\_{i}$ 3 .$ \bigcup\_{n=1}^{5}F\_{i}$ 4. $\bigcup\_{n=1}^{\infty }F\_{i}$ (4marks)
5. (a) Prove that an arbitrary collection of open sets is open. (4marks)

(b) Prove that a finite intersection of open sets is open, hence pick a counter example to verify that an arbitrary intersection of open sets is not necessarily open. (5marks)