JOMO KENYATTA UNIVERSITY
OF
AGRICULTURE AND TECHNOLOGY

University Examinations
STA 3114 -Survival Data Analysis – CAT

Date; **Time**; 3 Weeks

Instructions: Attempt ALL questions.

Question 1: (30 marks)

(a) Define the following terms as used in Survival Analysis:
(i) Survivorship function, (ii) Hazard function, (iii) Right censoring,
(iv) Left Censoring. (8 mks)

(b) Show that if the Hazard function has the form $\alpha\beta(\alpha t)^{\beta-1}\exp\left[(\alpha t)^{\beta}\right]$, then the Survivorship function is $\exp\left\{-\left[\exp(\alpha t)^{\beta}-1\right]\right\}$. (6 mks)

(c) Show that if T has a Weibull distribution given by $f(t)=\alpha\beta(\alpha t)^{\beta-1}\exp\left((-\alpha t)^{\beta}\right)$ with the scale parameter α and the index β, then T^{β} has an exponential distribution with parameter α^{β}.
(4 mks)

(d) Most Actuaries still use the Exponential Distribution of the form $f(t)=\begin{cases}\lambda e^{-\lambda t}, & t\geq 0\\ 0, & elsewhere\end{cases}$ as a premise for follow up studies. Derive the Survival function, the Hazard function and the Cumulative Hazard function, and give a comment regarding the structure of the Hazard function as is implied in Actuarial studies. (7 mks)

(e) Consider a discrete random variable T with atoms at 0, 1,2,3,4,…by first expressing the mean time to failure (Life Expectancy) as a function of the Survival function, obtain the life Expectancy of the following data.
(2 mks)

j	0	1	2	3	4	5
F_j	0.2	0.1	0.3	0.5	0.6	0.8

Question 2: (20 marks)

(a) Consider a continuous random variable T with PDF $f(t,\theta)$ and the Survivorship function $F(t,\theta)$ with $\theta=(\omega,\lambda)$ where ω is the parameter of interest while λ is just a nuisance parameter. Let $x_1,x_2,x_3,x_4,x_5,\ldots,x_n$ be observed values of T with some of them censored. Obtain the log-likelihood function in terms of the hazard function and the observed values of T. (12 mks)

(b) The following data give remission times (in weeks) of treated Leukemia patients. By carrying out one iteration, find the maximum likelihood estimates of the parameters when fitting a Weibull Distribution. (8 mks)

Question 3: (20 marks)

(a) Consider a discrete point random variable T. Show that in the usual Actuarial notations, $F(t)=\prod_{u_j<t}(1-h_j)$ and hence derive the Kaplan – Meir Estimator and obtain its asymptotic Variance as is mainly applicable to Actuaries. (10 mks)

(b) The following are times (in years) taken by10 Actuarial students to qualify as Actuaries after the start of the professional papers. 9, 13, 13*, 18, 23, 28, 31, 34, 45*, 48. Calculate the Kaplan-Meir estimator and comment on it with regard to this performance. (10 mks)

Question 4: (20 marks)

(a) Differentiate between a proportional hazards model and an accelerated life model and give a condition under which the two models coincide. (12 mks)

(b) The following are failure times of two groups of Asthma patients. Group 1 consists of patients who received the drug coded 10-mp while group 0 is the control group (did not receive the treatment). Assuming the exponential distribution for failure time, test the hypothesis that 10-mp treatment is not effective for Asthma at 95% level of significance. (8 mks)

Group 0	1	3	5	6*	12
Group 1	9	14	15	15*	30*

Question 5: (20 marks)

(a) Derive the general non-parametric test for comparing two survival functions from the perspective of a 2x2 contingency table, and hence obtain the proportional hazards (Mantel-Haenzel) test and the generalized Wilcoxon test. (10 mks)

(b) The following data provide 5 survival times under treatments A and B. use it to calculate the Mantel-Haenzel Statistic (10 mks)

Treatment A	3	5	7	9*	18
Treatment B	12	19	20	20*	33*